ZAINEMATICS

FOR THE LOVE OF MATHS

A - Level

M1 Notes

By

Zain Afaq

Compiled By Rafay Mushtaq

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Forces & Equilibrium

M1

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M1

Welcome!

M1 contains diff and integration of P1 only.

M1 | P4 | (50 Marks) (1h 15m) (7 to 8 Questions)

(VERY EASY PAPER EVEN FOR NON-SCIENCE PPL.)

HIGH PERCENTILE : 42 – 46 out of 50.

(TOTAL 12 WORKINGS)

FORCES

PUSH/PULL ⟶ FORCE → UNITS (NEWTON)

SPECIAL FORCE: WEIGHT

~~75kg~~ → Mass

Pull of gravity on an object.

$w = 75(10)$

$W = 750\,N$

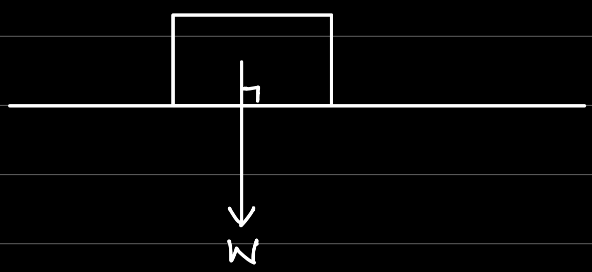
$$\boxed{W = mg}$$

mass
m(kg)   g = 10 .

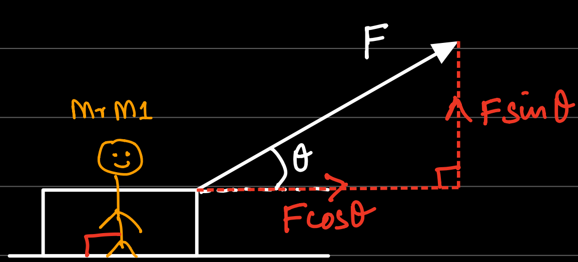
g = acceleration due to gravity = 10 N/kg
10 m/s²

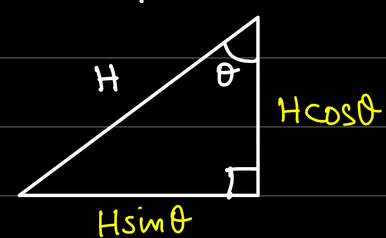
(In whole M1 , g = 10) (In Physics g = 9.81)

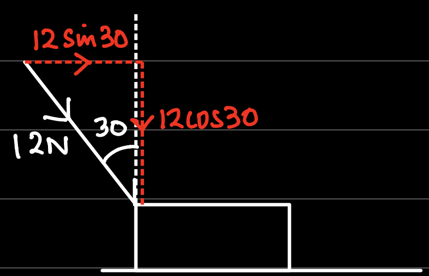
DIRECTION : TOWARDS GROUND MAKING 90°
WITH GROUND (HORIZONTAL) .
Line of weight starts from
centre of object .

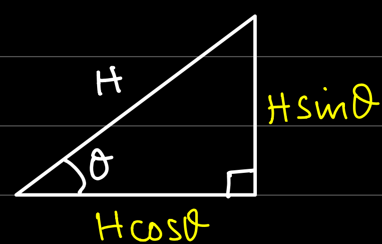


COMPONENTS OF A FORCE :-

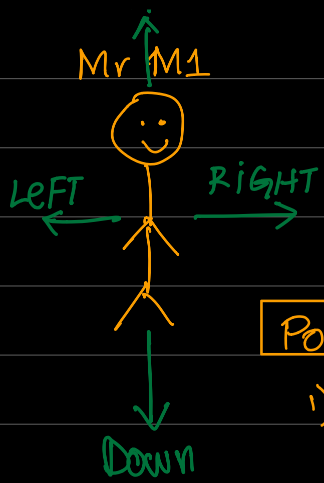








UP



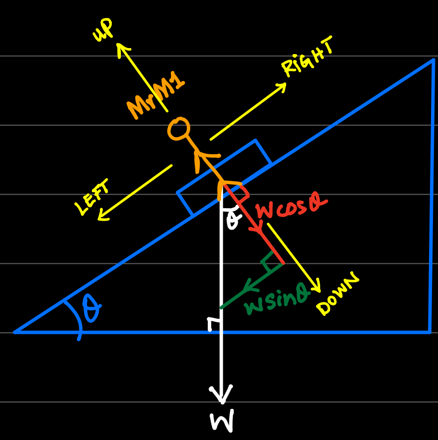
Can only understand forces which are UP/DOWN/LEFT/RIGHT.

POSITION OF M1

1) STANDS ON POINT OF CONTACT BETWEEN OBJECT & SURFACE.
2) ALWAYS STANDS SUCH THAT HE MAKES 90° TO SURFACE.

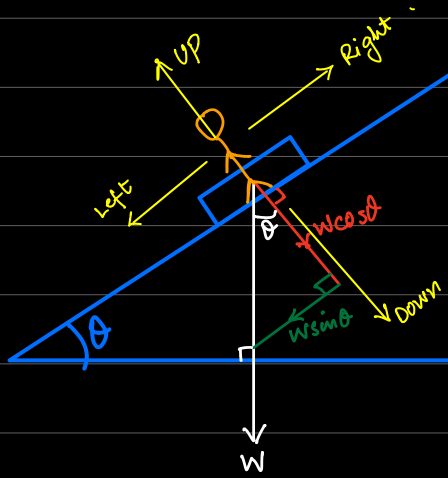
IN COMPLETE M1 THE WORDS UP/ DOWN / LEFT / RIGHT ARE USED FROM MR M1'S PERSPECTIVE.

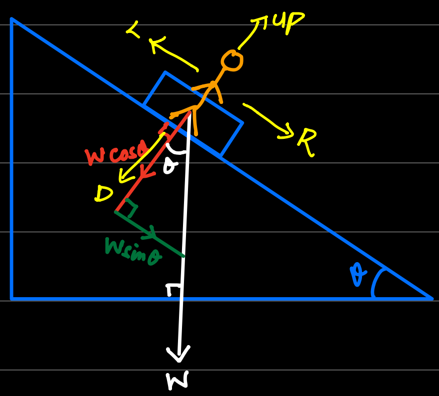
INCLINED PLANES:

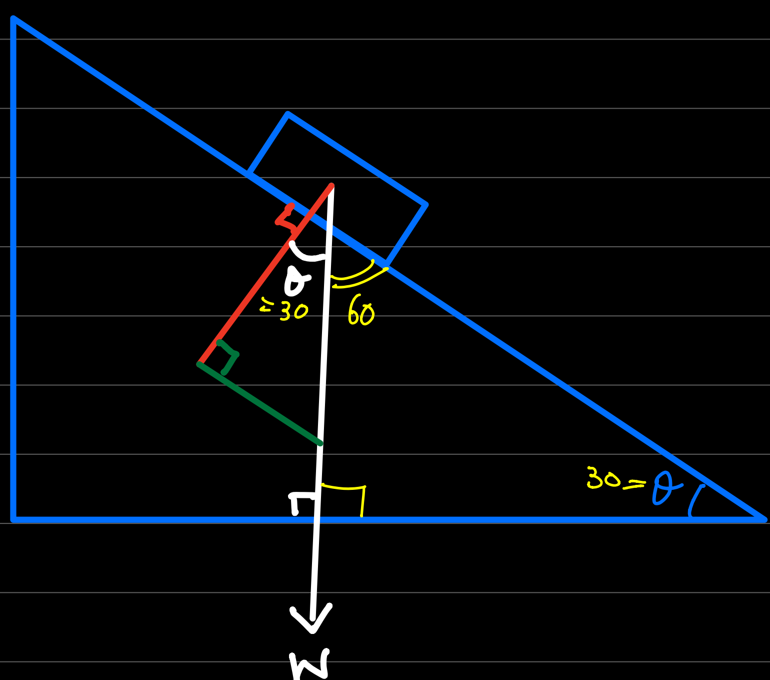


THREE LINES.

1) Weight (90° to ground)
2) 90° TO PLANE
3) PARALLEL TO PLANE.


UP
Right
Left
Wcosθ
θ
Down
Wsinθ
θ
W


L
UP
R
Wcosθ
θ
D
Wsinθ
θ
W


θ
=30
60
30=θ
W

ZAINEMATICS

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Tension

M1

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TENSION

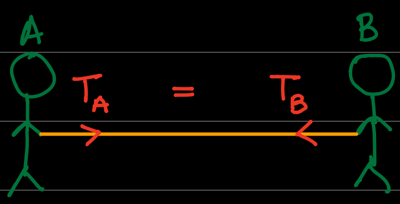
FORCE APPLIED BY THE ROPE/STRING ON AN OBJECT.

Slack (No Tension) | Taut (There is tension)

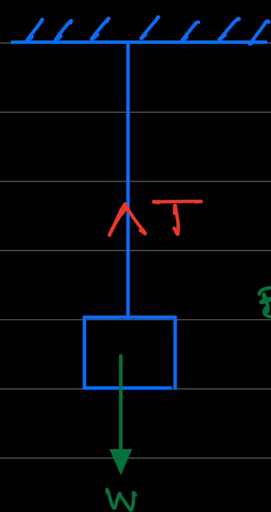
DIRECTION OF TENSION:

ACTS AWAY FROM POINT OF OBSERVATION

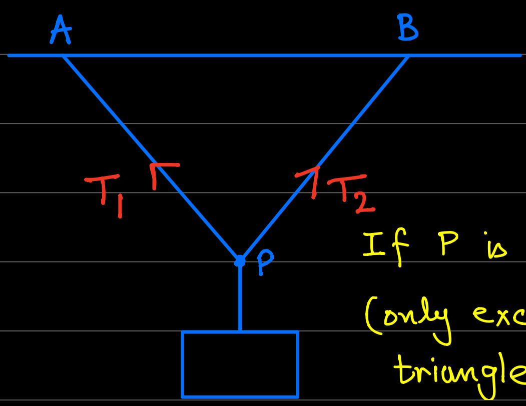
POINT OF OBSERVATION IS MENTIONED IN QUESTION.



Rope is Taut.



Box is AT REST.

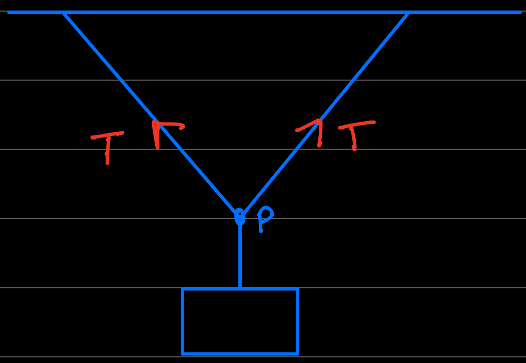


P is a fixed point.

P is point of observation.

If P is fixed T_1 and T_2 are different

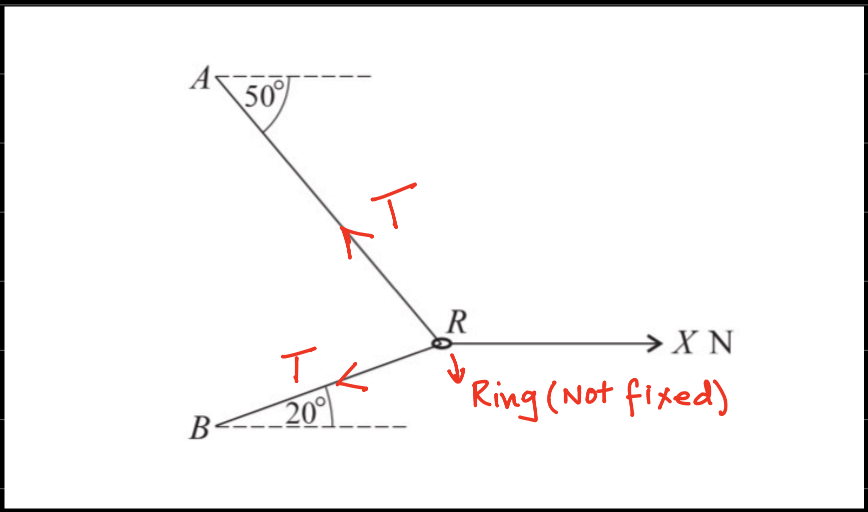
(only exception is if there is an isosceles triangle. In that case only $T_1 = T_2$).



P is a smooth ring.
String is threaded through the ring.
(Ring can slip).

P = point of observation.

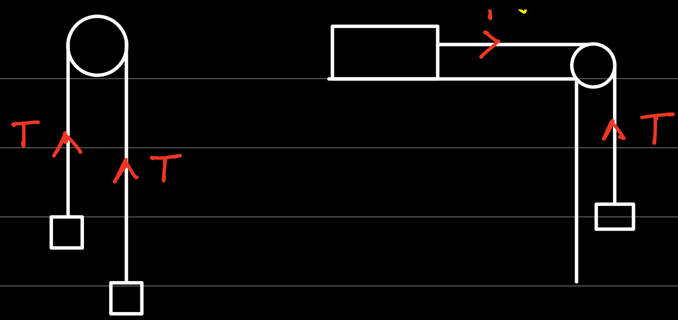
IF POINT P can slip, BOTH TENSIONS ARE ALWAYS SAME REGARDLESS OF SHAPE OF DIAGRAM.



PULLEYS / PEG :-

IN ANY SHAPE, TENSION IN A PULLEY SYSTEM IS SAME AND ACTS TOWARDS THE PULLEY.

Even if weights are different, objects are at rest or are moving.



Objects are points of observation here.

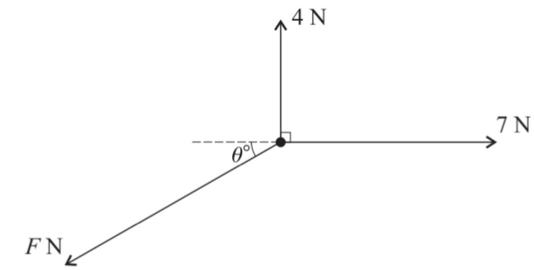
$\boxed{\text{WORKING \# 1}}$ EQUILIBRIUM

(BALANCED) (STATIONARY) (REST)

UP = down Left = Right.

(UP, DOWN, LEFT, RIGHT is FROM Mr M1's PERSPECTIVE)

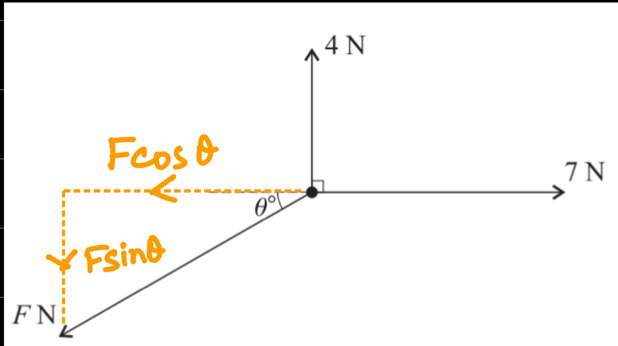
8



A particle is in $\boxed{\text{equilibrium}}$ on a smooth horizontal table when acted on by the three horizontal forces shown in the diagram.

NO FRICTION.

(i) Find the values of F and θ. [4]



up = down

$4 = F\sin\theta$

$\boxed{F\sin\theta = 4}$

Left = Right.

$\boxed{F\cos\theta = 7}$

THIS TYPE OF SIMULTANEOUS IS SPECIAL.

$\left\{\begin{array}{l} F\sin\theta = \square \\ F\cos\theta = \square \end{array}\right\}$ THIS WORKING COMES IN P3 AND M1

STEP 1

$$\frac{\cancel{F}\sin\theta}{\cancel{F}\cos\theta} = \frac{4}{7}$$

$$\tan\theta = \underline{4}$$

STEP 2 SQUARE BOTH EQUATIONS AND ADD.

$$F^2\sin^2\theta = 16$$

$$+ \quad F^2\cos^2\theta = 49$$

$$F^2\sin^2\theta + F^2\cos^2\theta = 16 + 49$$

7

$\theta = \tan^{-1}\left(\frac{4}{7}\right)$

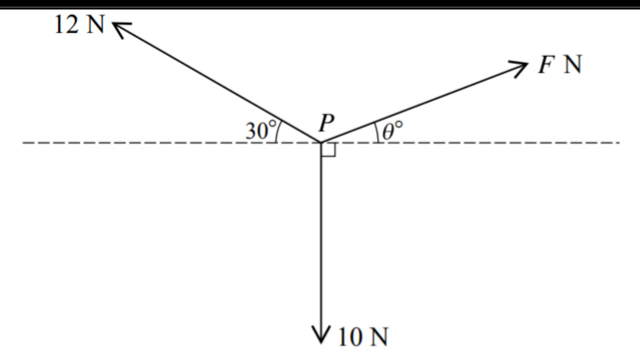
$\boxed{\theta = 29.74}$

$F^2(\sin^2\theta + \cos^2\theta) = 65$

$F^2(1) = 65$

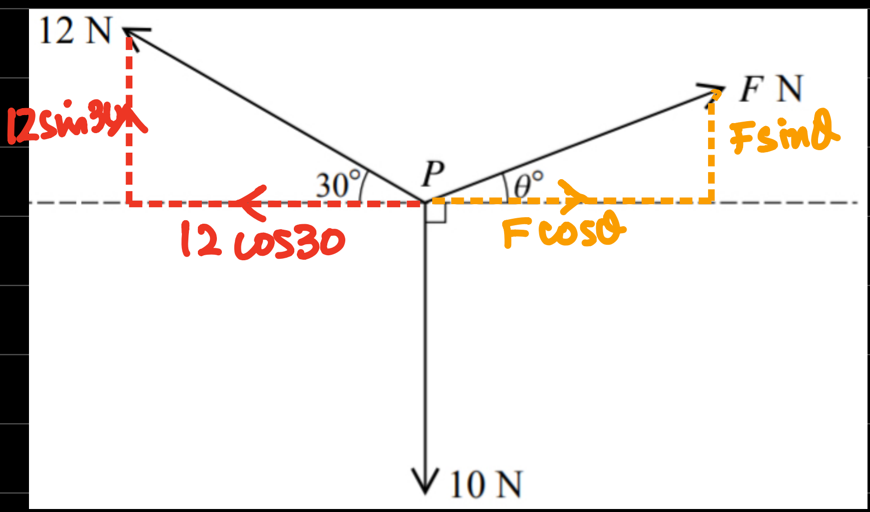
$F^2 = 65$

$\boxed{F = \sqrt{65}} = 8.0622.$



The three ~~coplanar~~ forces shown in the diagram act at a point P and are in equilibrium.

(i) Find the values of F and θ. [6]



up = down

$F\sin\theta + 12\sin 30 = 10$

$\boxed{F\sin\theta = 4}$

Left = Right

$12\cos 30 = F\cos\theta$

$F\cos\theta = 12\left(\sqrt{3}\right)$

(2 /

$\boxed{F\cos\theta = 6\sqrt{3}}$

$(6\sqrt{3})^2$

$6^2(\sqrt{3})^2$

$36(3)$

108

STEP 1

$$\frac{F\sin\theta}{F\cos\theta} = \frac{4}{6\sqrt{3}}$$

$$\tan\theta = \frac{2}{3\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{2}{3\sqrt{3}}\right)$$

$$\boxed{\theta = 21.05}$$

STEP 2

$$F^2\sin^2\theta = 16$$

$$+\ F^2\cos^2\theta = 108$$

$$F^2\sin^2\theta + F^2\cos^2\theta = 124$$

$$F^2(\sin^2\theta + \cos^2\theta) = 124$$

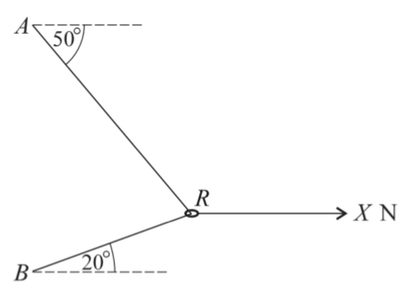
$$F^2(1) = 124$$

$$F^2 = 124$$

$$F = \sqrt{124}$$

$$\boxed{F = 11.1355}$$

1



$W = mg = 0.8 \times 10 = 8N$

A ~~light inextensible~~ string has its ends attached to two fixed points A and B, with A vertically above B. A smooth ring R, of mass 0.8 kg, is threaded on the string and is pulled by a horizontal force of magnitude X newtons. The sections AR and BR of the string make angles of 50° and 20° respectively with the horizontal, as shown in the diagram. The ring rests in equilibrium with the string taut. Find

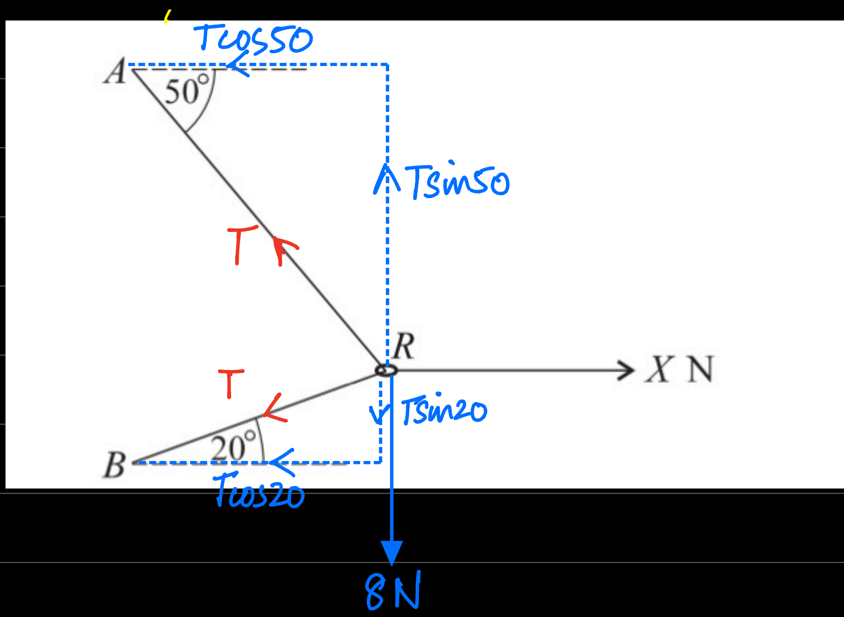
point of observation.

(i) the tension in the string, [3]

(ii) the value of X. [3]

NEVER MARK ANY FORCES ON THE DIAGRAM OF QUESTION.

DRAW A HUGE DIAGRAM FOR YOURSELF.



up = down

$T\sin 50 = 8 + T\sin 20$

$T\sin 50 - T\sin 20 = 8$

$T(\sin 50 - \sin 20) = 8$

$T = \dfrac{8}{\sin 50 - \sin 20}$

$T = 18.87.$

Left = Right.

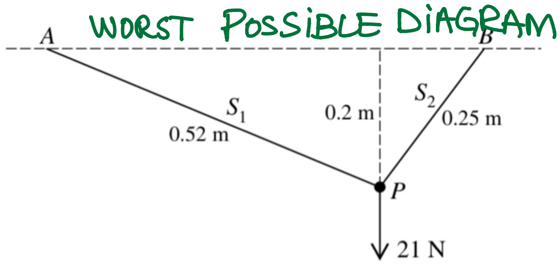
$T\cos 50 + T\cos 20 = X$

$T(\cos 50 + \cos 20) = X$

$18.87(\cos 50 + \cos 20) = X$

$X = 29.86.$

26

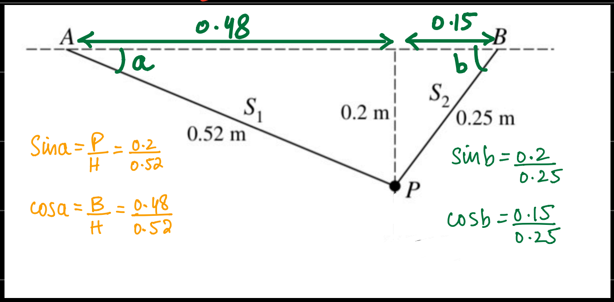


WORST POSSIBLE DIAGRAM. = You have both distances and forces on same diagram.

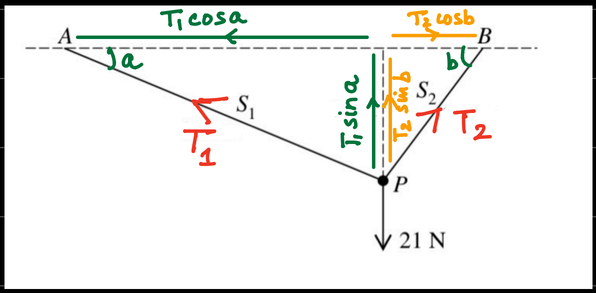
A particle P of weight 21 N is attached to one end of each of two ~~light inextensible~~ strings, S_1 and S_2, of lengths 0.52 m and 0.25 m respectively. The other end of S_1 is attached to a fixed point A, and the other end of S_2 is attached to a fixed point B at the same horizontal level as A. The particle P hangs in equilibrium at a point 0.2 m below the level of AB with both strings taut (see diagram). Find the tension in S_1 and the tension in S_2. [6]

EXAM TIP: DRAW SEPERATE DIAGRAMS FOR DISTANCES AND FORCES.

DISTANCES



FORCES.



STAY IN FRACTIONS THROUGHOUT QUESTION.

up = down

$$T_1 \sin a + T_2 \sin b = 21$$

$$T_1 \left(\frac{0.2}{0.52}\right) + T_2 \left(\frac{0.2}{0.25}\right) = 21$$

$$\boxed{\frac{5}{13} T_1 + \frac{4}{5} T_2 = 21}$$

Left = Right.

$$T_1 \cos a = T_2 \cos b$$

$$T_1 \left(\frac{0.48}{0.52}\right) = T_2 \left(\frac{0.15}{0.25}\right)$$

$$\frac{\cancel{12}^{4}}{13} T_1 = \frac{\cancel{3}^{1}}{5} T_2$$

$$\boxed{T_1 = \frac{13}{20} T_2}$$

Solving Simultaneously.

$$\frac{5}{13}\left(\frac{13}{20}\right) T_2 + \frac{4}{5} T_2 = 21$$

$$\frac{1}{4} T_2 + \frac{4}{5} T_2 = 21$$

$$\frac{21}{20} T_2 = 21$$

$$\boxed{T_2 = 20}$$

$$T_1 = \frac{13}{20} \times 20$$

$$\boxed{T_1 = 13}$$

ZAINEMATICS

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Friction

M1

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FRICTION

$F = \mu R$

F → Maximum Friction.
μ → Cofficient of friction.
R → Normal Reaction force

This formula can only be applied if body is **ABOUT TO MOVE** OR **MOVING**
This formula cannot be used if a body is at **REST**

TYPE OF SURFACE

Cofficient of Friction (μ)

Slippery surface ($\mu \downarrow$) (Friction$\downarrow$)

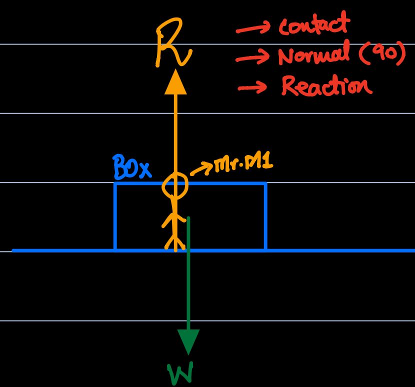
Rough surface ($\mu \uparrow$) (Friction$\uparrow$)

CONTACT FORCE	R
NORMAL FORCE	
REACTION FORCE	

Always Find R using

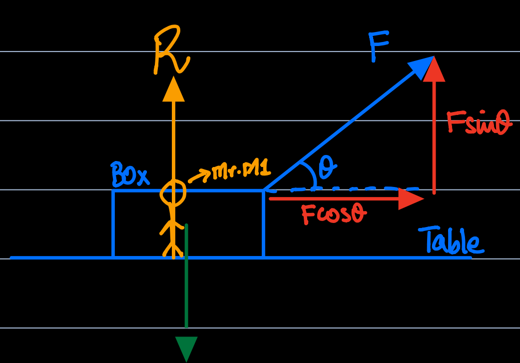
UP = DOWN

$\boxed{R}$ acts towards head of Mr M1.



up = down

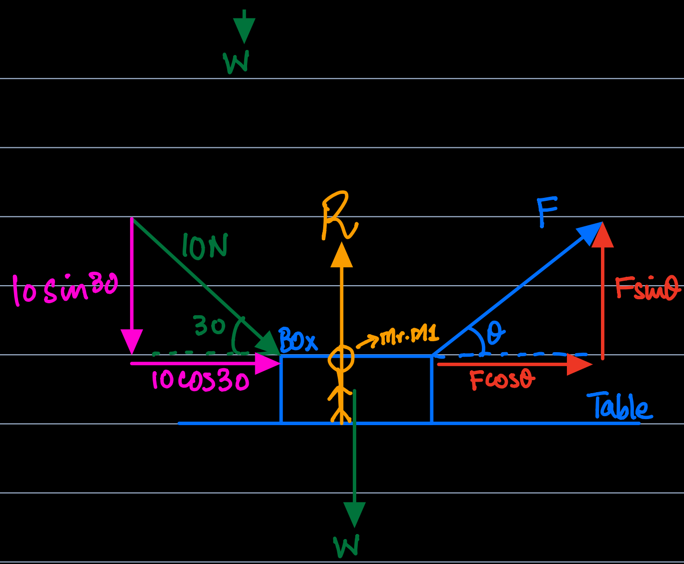
$R = W$



up = down

$R + F\sin\theta = W$

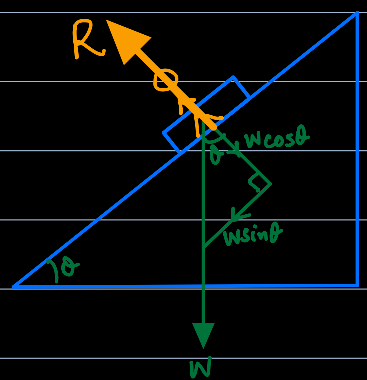
$R = W - F\sin\theta$



up = down

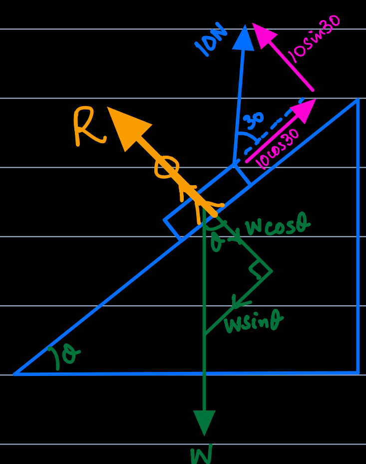
$R + F\sin\theta = 10\sin 30 + W$

$R = 10\sin 30 + W - F\sin\theta$



up = down

$R = W\cos\theta$



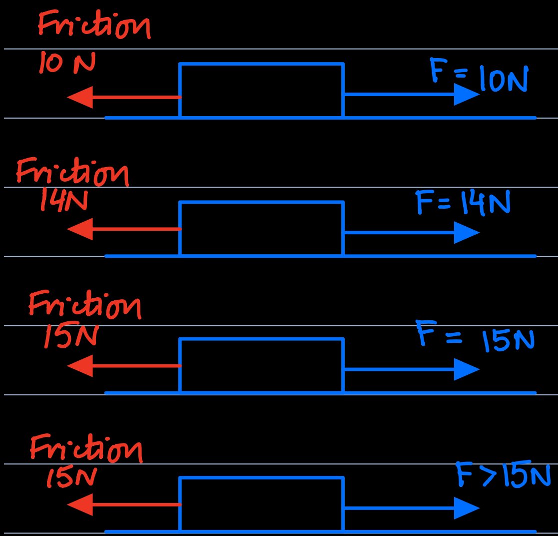
up = down

$R + 10\sin 30 = W\cos\theta$

$R = W\cos\theta - 10\sin 30$

THOUGHT EXPERIMENT

Lets suppose, Max friction between box and surface is 15N.



Friction		
10 N	F = 10N	REST
14N	F = 14N	REST
15N	F = 15N	ABOUT TO MOVE (LIMITING EQUILIBRIUM)
15N	F > 15N	MOVE

At rest, Friction is not constant. It depends upon how much force is applied on object.

Maximum Friction

$$F = \mu R$$

FRICTION

REST | ABOUT TO MOVE (LIMITING EQUILIBRIUM) | MOVE

You cannot apply $F = \mu R$

we simply label it "Friction" and apply up = down

ABOUT TO MOVE and MOVE: $F = \mu R$

$\boxed{\text{left} = \text{Right.}}$

LIMITING EQUILIBRIUM

ABOUT TO MOVE/SLIP.

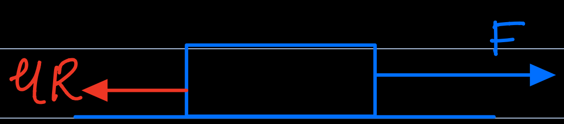
STEP1 MARK THE DIRECTION OF FRICTION AGAINST DIRECTION OF MOTION AND LABEL IT

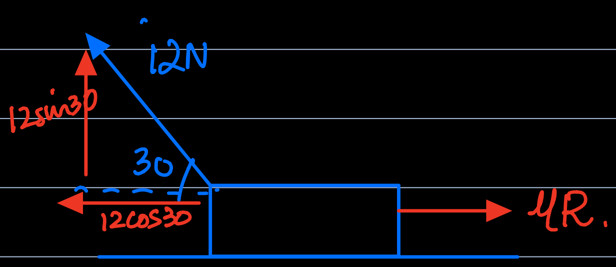
$F = \mu R$

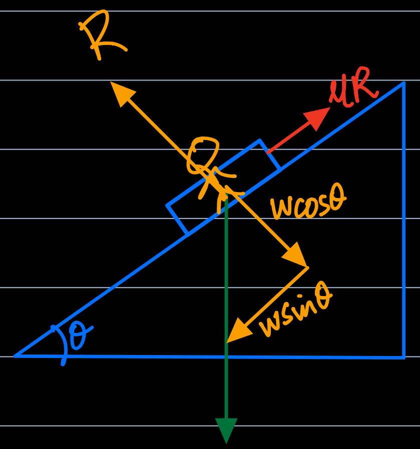
STEP2 Apply up = down

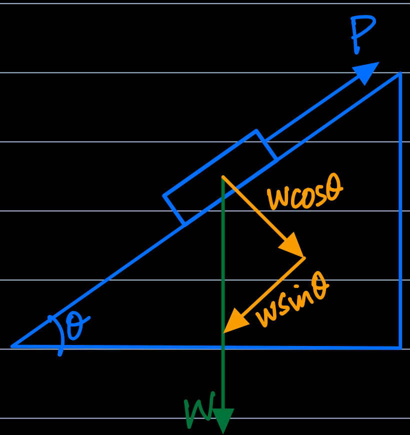
left = Right.

BOX IS ABOUT TO MOVE.









Force P pulls the box upwards.
Box is about to move.

In this case we cannot determine the direction of motion of box.
Hence we cannot label friction.

In this case there are two options:

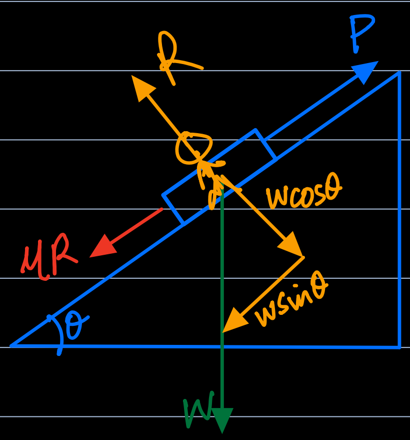
1) Question will tell you direction of motion (4 marks)

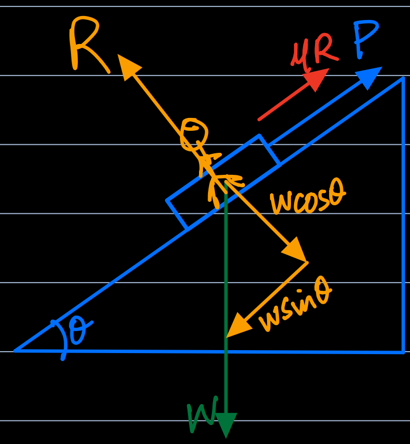
2) Question will ask for two values of unknown.

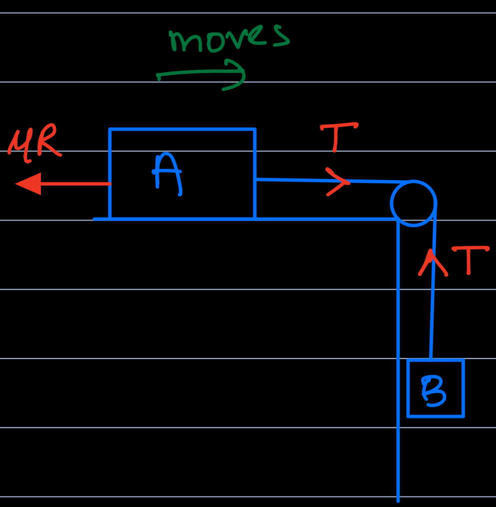
(8marks)

You will assume two cases.

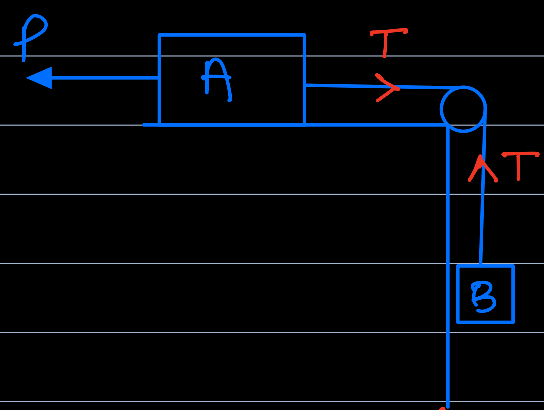
Case 1	Case 2
box moves up	box moves down







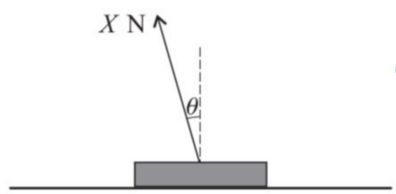
System is released from rest and Box A is about to move.



System is released from rest and Box A is about to move.

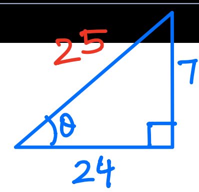
We cannot determine direction of motion.

5



$\sin\theta = \frac{7}{25}$

$\cos\theta = \frac{24}{25}$



$W = 3200N$ we cannot use $F = \mu R$.

A stone slab of mass 320 kg rests in equilibrium on rough horizontal ground. A force of magnitude X N acts upwards on the slab at an angle of θ to the vertical, where $\tan\theta = \frac{7}{24}$ (see diagram).

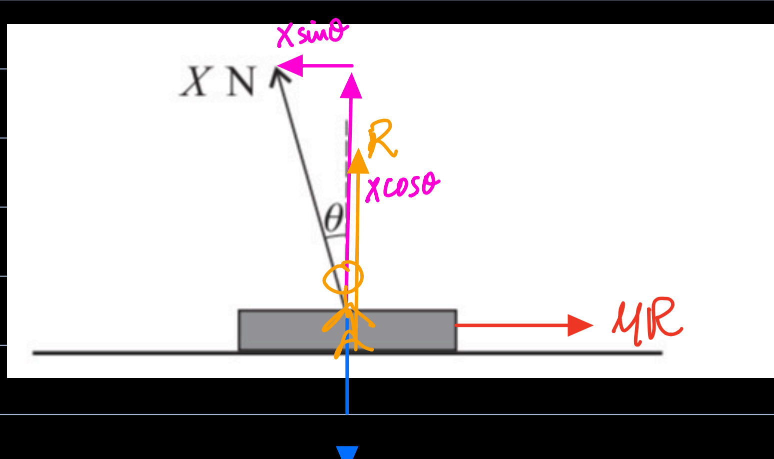
R

(i) Find, in terms of X, the normal component of the force exerted on the slab by the ground. [3]

(ii) Given that the coefficient of friction between the slab and the ground is $\frac{3}{8}$, find the value of X for which the slab is about to slip. (Limiting equilibrium) [3]

$\mu = \frac{3}{8}$

Now Label friction as $F = \mu R$



3200N

(i) up = down

$R + X\cos\theta = 3200$

$R = 3200 - X\cos\theta$

$$\boxed{R = 3200 - \frac{24}{25}X}$$

(ii) Left = Right

$X\sin\theta = \mu R.$

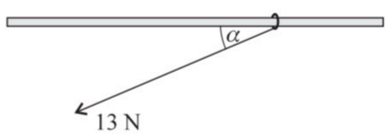
$X\left(\frac{7}{25}\right) = \frac{3}{8}\left(3200 - \frac{24}{25}X\right)$

$\frac{7}{25}X = 1200 - \frac{9}{25}X$

$\frac{16}{25}X = 1200$

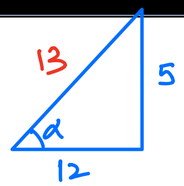
$X = 1875\text{N}.$

1



$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$



W = 11N

Friction.

A ring of mass 1.1 kg is threaded on a fixed rough horizontal rod. A ~~light~~ string is attached to the ring and the string is pulled with a force of magnitude 13 N at an angle α below the horizontal, where $\tan\alpha = \frac{5}{12}$ (see diagram). The ring is in equilibrium.

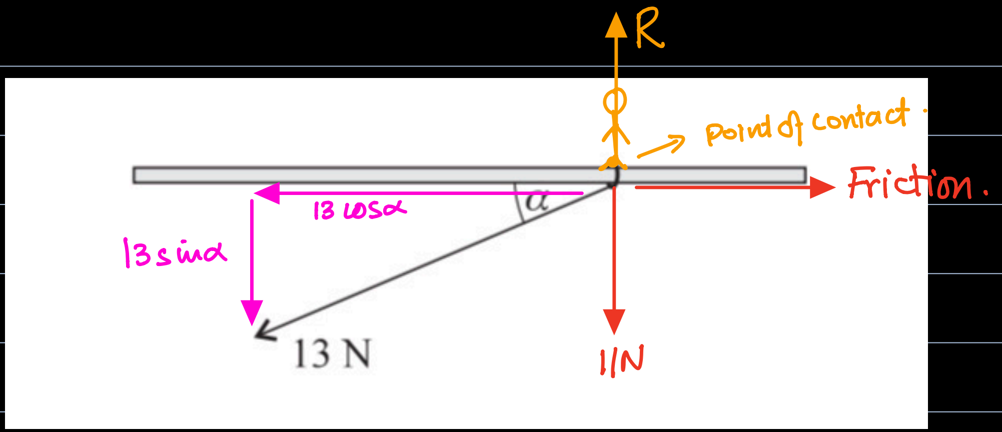
(i) Find the frictional component of the contact force on the ring. [2]

R

(ii) Find the normal component of the contact force on the ring. [2]

(iii) Given that the equilibrium of the ring is limiting, find the coefficient of friction between the ring and the rod. [1]

Friction = μR μ



(i) Left = Right

$13\cos\alpha$ = Friction

$13\left(\frac{12}{13}\right)$ = Friction

Friction = 12 N

(ii) up = down

$R = 11 + 13\sin\alpha$

$R = 11 + 13\left(\frac{5}{13}\right)$

$R = 16$ N

(iii)

$$F = \mu R$$

$$12 = \mu(16)$$

$$\mu = \frac{3}{4}$$

ZAINEMATICS

FOR THE LOVE OF MATHS

Work, Power & Energy M1

Compiled by Rafay Mushtaq

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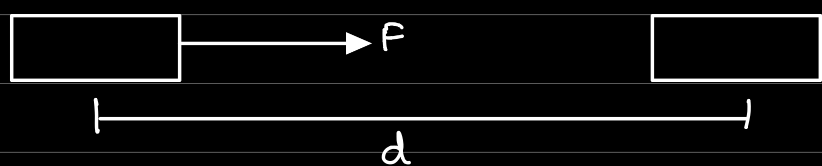
WORK - POWER - ENERGY.

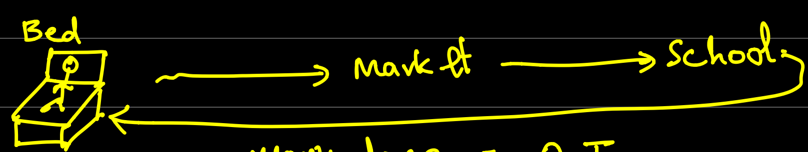
$\boxed{\text{WORK}}$ work is said to be done when a force displaces an object in its direction

parallel



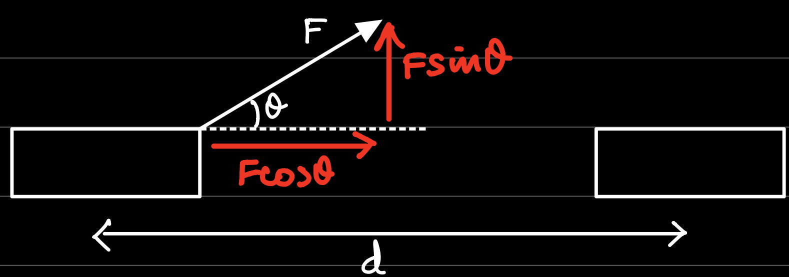
WORK DONE = $\underbrace{F \times d}_{\text{must be parallel to each other}}$

UNIT = Joules (J)



work done = 0 J

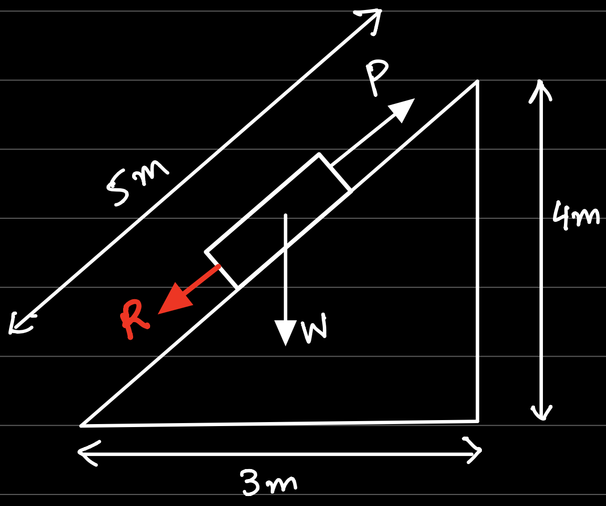
Start point = end point $\rightarrow$ disp = 0



$$W = (F\cos\theta) \times (d)$$

FORCE AND DISPLACEMENT MUST BE PARALLEL.

ARROWS CAN BE POINTING AWAY.



(i) Workdone by Force P in pulling object from bottom to top.

$W = F \times d$

$W = (P) \times (5)$

(ii) Work done against gravity.

weight

$W = F \times d$

$W = (W) \times (4)$

(iii) Workdone against resistive forces.

$W = F \times d$

$W = (R) \times (5)$

$\boxed{\text{ENERGY}}$

GRAVITATIONAL POTENTIAL ENERY	KINETIC ENERGY.
Due to height of object	Due to speed of object.
$PE = mgh$	$KE = \frac{1}{2} m v^2$
Units = Joules (J)	Units = Joules (J)

POWER

$$\boxed{\text{Power} = \frac{\text{Workdone/Energy}}{\text{time}}} \quad \frac{J}{s}$$

$$P = \frac{W}{t} = \frac{F \times d}{t} = F \times v$$

$$\boxed{P = F\,v}$$

UNITS : WATTS (W) OR J/s

TRICKS :

1) KW means 1000 W
   5 KW means 5000 W
   P KW means 1000 P W

2) They use units
   J/s or $J\,s^{-1}$
   instead of Watts.

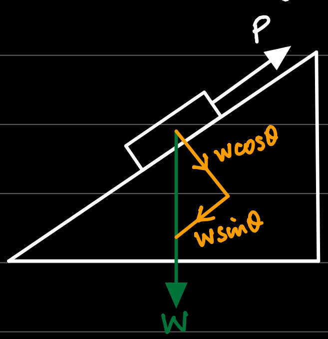
F is different in all of these four formulas.

$F = ma$	$W = F \times d$	$P = F \times v$	$F = \mu R$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Fwd - Bwd	Desired force whose work done is to be found.	Driving force	Maximum Friction

FORWARD FORCE

Any force acting in the direction of motion

Box is moving upwards



Fwd force = P

Bwd force = Wsinθ

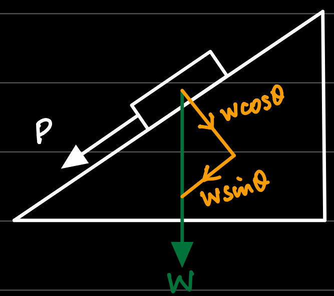
DRIVING FORCE

→ Has to be an external force

→ Acts in same direction as direction of motion

→ Cannot include weight(W) or components (Wsinθ, Wcosθ)

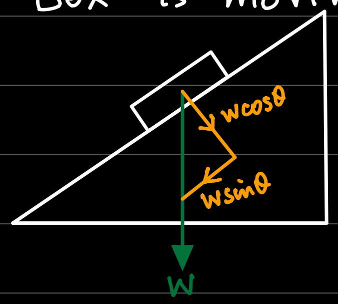
Box is moving downwards.



Fwd Force = P + W sinθ

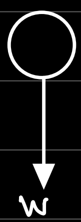
Driving Force = P

Box is moving down.



Fwd = $W\sin\theta$

Driving force = 0



Ball is falling freely under gravity.

Fwd force = W

Driving force = 0

$\boxed{\text{TYPE 1}}$ QUESTION WILL DISCUSS

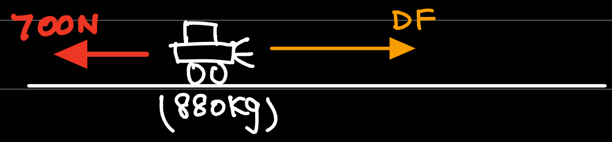
$\boxed{\text{FORCE}}$, $\boxed{\text{ACCELERATION}}$, $\boxed{\text{POWER}}$

$$\boxed{\text{Fwd} - \text{Bwd} = ma}$$

$$\boxed{\begin{aligned} &P = \overset{\text{Driving}}{F}v \\ &P = (DF)v \end{aligned}}$$

Apply both of these at same time and find unknown.

46 A car of mass 880 kg travels along a straight horizontal road with its engine working at a constant rate of P W. The resistance to motion is 700 N. At an instant when the car's speed is 16 m s^{-1} its acceleration is 0.625 m s^{-2}. Find the value of P. [4]



Power = P

acc = 0.625

v = 16

Fwd − Bwd = ma

DF − 700 = 880(0.625)

DF = 1250

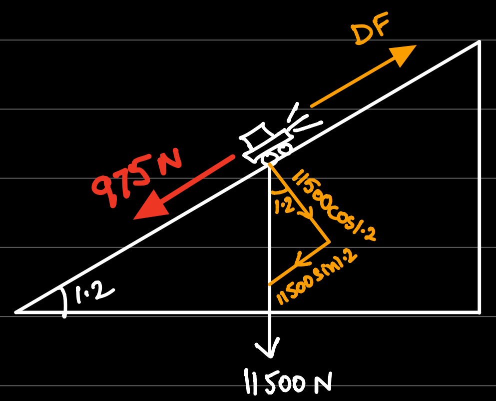
P = (DF)(v)

P = (1250)(16)

P = 20,000 W

W = 11500

25 A car of mass 1150 kg travels up a straight hill inclined at 1.2° to the horizontal. The resistance to motion of the car is 975 N. Find the acceleration of the car at an instant when it is moving with speed 16 m s^{-1} and the engine is working at a power of 35 kW. [4]



a = ?

v = 16

P = 35 KW = 35000 W

Fwd − Bwd = ma

DF − 975 − 11500 sin1.2 = 1150a

P = (DF) v

35000 = (DF)(16)

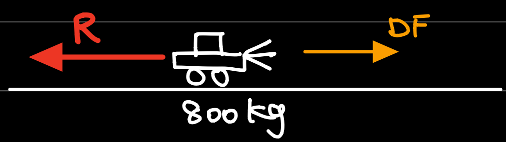
DF = 2187.5

2187.5 − 975 − 11500 Sin1.2 = 1150a

a = 0.8449.

Power = 1000 P

67 A car has mass 800 kg. The engine of the car generates constant power P kW as the car moves along a straight horizontal road. The resistance to motion is constant and equal to R N. When the car's speed is $14\ \mathrm{m\,s^{-1}}$ its acceleration is $1.4\ \mathrm{m\,s^{-2}}$, and when the car's speed is $25\ \mathrm{m\,s^{-1}}$ its acceleration is $0.33\ \mathrm{m\,s^{-2}}$. Find the values of P and R. [6]



$v = 14$, $a = 1.4$

Power $= 1000P$

Fwd − Bwd = ma

$DF - R = 800(1.4)$

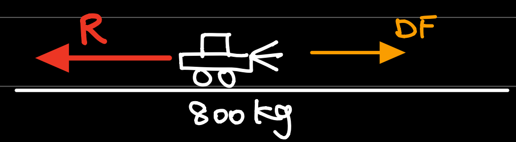
$\frac{1000P}{14} - R = 1120$

$$\boxed{R = \frac{1000P}{14} - 1120}$$

$P = (DF)v$

$1000P = DF(14)$

$$\boxed{DF = \frac{1000P}{14}}$$



$v = 25$, $a = 0.33$

Power $= 1000P$

Fwd − Bwd = ma

$DF - R = 800(0.33)$

$40P - R = 264$

$$\boxed{R = 40P - 264}$$

$P = (DF)v$

$1000P = DF(25)$

$DF = \frac{1000P}{25}$

$$\boxed{DF = 40P}$$

NOW SOLVE SIMULTANEOUS.

$$\frac{1000P}{14} - 1120 = 40P - 264$$

$$\frac{1000P}{14} - 40P = 1120 - 264$$

$$\frac{220}{7}P = 856$$

$P = 27.236$, $R = 40(27.236) - 264$

$R = 825.45$

TYPE 2 | WORK DONE or ENERGY

MARKS : Range from 1 Mark to 9 marks.

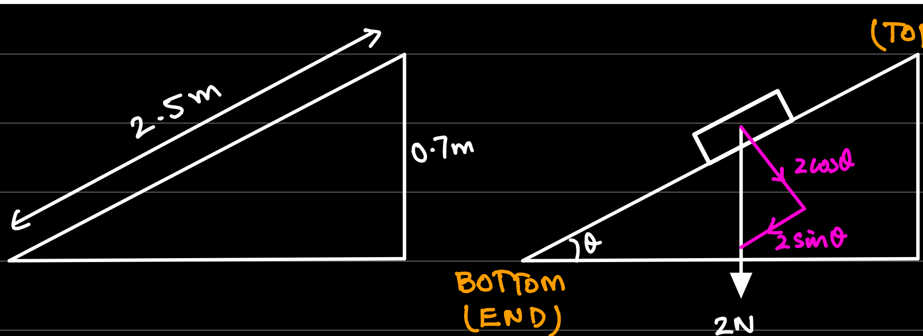
BEFORE = AFTER

$$\begin{pmatrix} PE = \\ KE = \\ \text{Workdone by Driving force} = \end{pmatrix} = \begin{pmatrix} PE = \\ KE = \\ \text{Work done against friction} = \end{pmatrix}$$

1 The top of an inclined plane is at a height of 0.7 m above the bottom. A block of mass 0.2 kg is released from rest at the top of the plane and slides a distance of 2.5 m to the bottom. Find the kinetic energy of the block when it reaches the bottom of the plane in each of the following cases:

(i) the plane is smooth, (No friction) [2]

(ii) the coeff cient of friction between the plane and the block is 0.15. [5]



$F_{wd} = 2\sin\theta$

DF = 0

BEFORE = AFTER

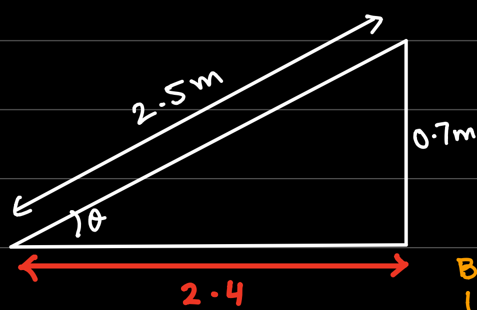
TOP = BOTTOM

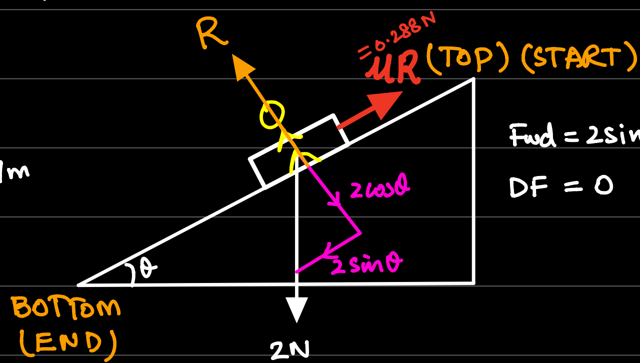
$$\begin{pmatrix} PE = mgh = (0.2)(10)(0.7) = 1.4 \\ KE = 0 \\ \text{Workdone by Driving force} = 0 \end{pmatrix} = \begin{pmatrix} PE = 0 \\ KE = ? \\ \text{Work done against friction} = 0 \end{pmatrix}$$

$$1.4 + 0 + 0 = 0 + KE_B + 0$$

$$KE_B = 1.4\ \text{J}$$

(iii) PLANE IS ROUGH: $\mu = 0.15$





Fwd = 2sinθ

DF = 0

$$\sin\theta = \frac{0.7}{2.5} = \frac{7}{25}$$

$$\cos\theta = \frac{2.4}{2.5} = \frac{24}{25}$$

Apply up = down to find R

$R = 2\cos\theta$

$R = 2\left(\frac{24}{25}\right) = 1.92$

Apply $F = \mu R$ to find friction.

$F = (0.15)(1.92)$

$F = 0.288.$

BEFORE		AFTER
Top		Bottom
PE = $(0.2)(10)(0.7) = 1.4$	=	PE = 0
KE = 0		KE = ?
Workdone by Driving force = 0		Work done against friction = $(0.288) \times (2.5)$

$$1.4 + 0 + 0 = 0 + KE_B + (0.288)(2.5)$$

$$KE_B = 0.68 \text{ J}$$

ZAINEMATICS

FOR THE LOVE OF MATHS

Momentum
M1

Compiled by Rafay Mushtaq

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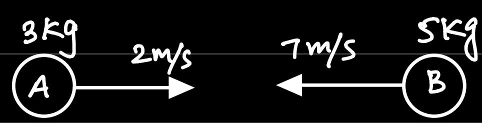
MOMENTUM = PRODUCT OF MASS AND VELOCITY.

$$\boxed{P = mv}$$

Units = kg.m/s or $kg\,ms^{-1}$

IMP: DIRECTION WITH VELOCITY.



$P_A = (3)(2)$
$= 6 \; kg\,ms^{-1}$

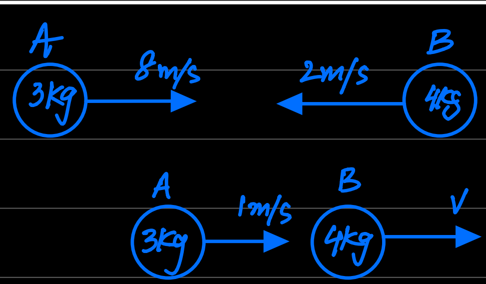
$P_B = (5)(-7)$
$-35 \; kg\,ms^{-1}$

COLLISIONS

TOTAL MOMENTUM BEFORE COLLISION = TOTAL MOMENTUM AFTER COLLISION

Q. Before

After.



, Find v.

TOTAL MOMENTUM BEFORE = TOTAL MOMENTUM AFTER

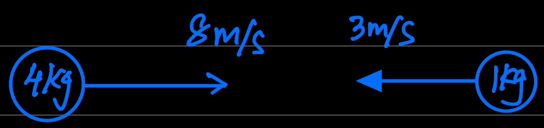
$$(3)(8) + (4)(-2) = (3)(1) + (4)(v)$$

$$24 - 8 = 3 + 4v$$

$$16 - 3 = 4v$$

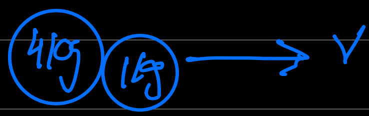
$v = 3.25$

Q:



After the collision, Both boxes coalesce,

Coalesce = Join together



TOTAL MOMENTUM BEFORE = TOTAL MOMENTUM AFTER.

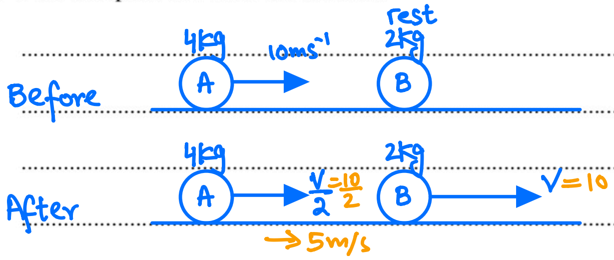
$(4)(8) + (1)(-3) = (4+1)(v)$

$32 - 3 = 5v$

$v = 5.8$

4 Small smooth spheres A and B, of equal radii and of masses 4 kg and 2 kg respectively, lie on a smooth horizontal plane. Initially B is at rest and A is moving towards B with speed $10\,\mathrm{m\,s^{-1}}$. After the spheres collide A continues to move in the same direction but with half the speed of B.

(a) Find the speed of B after the collision. [2]



$$\text{Before} = \text{After}$$

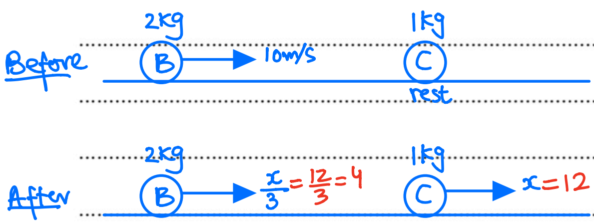
$$(4)(10) + (2)(0) = 4\left(\frac{v}{2}\right) + (2)(v)$$

$$40 = 2v + 2v$$

$$v = 10$$

A third small smooth sphere C, of mass 1 kg and with the same radius as A and B, is at rest on the plane. B now collides directly with C. After this collision B continues to move in the same direction but with one third the speed of C.

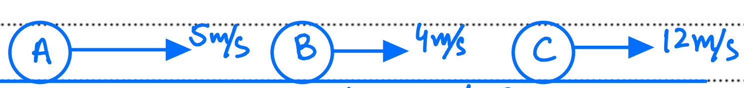
(b) Show that there is another collision between A and B. [3]



$$\text{Before} = \text{After}$$

$$(2)(10) + (1)(0) = 2\left(\frac{x}{3}\right) + 1(x)$$

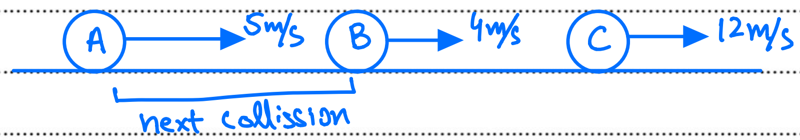
$$x = 12$$

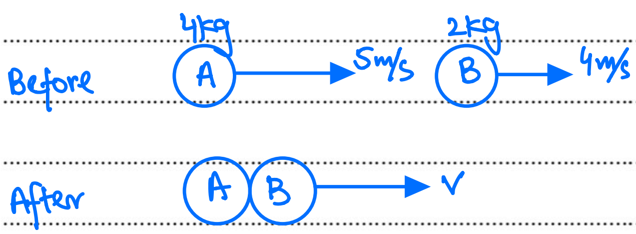


Since speed of A = 5 and speed of B = 4, A will catch up and there will be another collission.

(c) A and B coalesce during this collision.

Find the total loss of kinetic energy in the system due to the three collisions. [5]



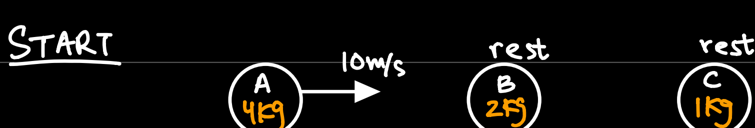


Before = After

$(4)(5)+(2)(4) = (4+2)v$

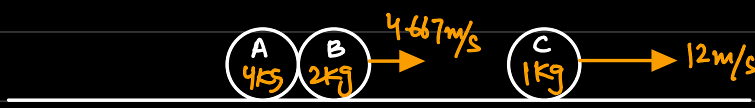
$28 = 6v$

$v = 4.667$



$$\text{TOTAL KE} = \frac{1}{2}(4)(10)^2 + \frac{1}{2}(2)(0)^2 + \frac{1}{2}(1)(0)^2 = 200\,J$$

END



$$\text{TOTAL KE} = \frac{1}{2}(4+2)(4.667)^2 + \frac{1}{2}(1)(12)^2 = 137.15\,J$$

$$\text{LOSS IN KE} = 200 - 137.15 = 62.85\,J.$$

ZAINEMATICS

FOR THE LOVE OF MATHS

Kinematics
M1

Compiled by Rafay Mushtaq

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$\boxed{\text{KINEMATICS}}$

(M1) (20 + marks)

DISTANCE

(cm, m, km)

DISPLACEMENT

(cm, m, km)

> SHORTEST DISTANCE BETWEEN **START** AND **END** POINT.

SPEED

(m/s or km/h)

VELOCITY.

(m/s or km/h)

> IF QUESTION USES TERM "DISPLACEMENT" OR "VELOCITY" BODY IS ALWAYS MOVING IN STRAIGHT LINE.

ACCELERATION Rate of change in speed.

$$a = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{time}}.$$

$$\boxed{a = \frac{v - u}{t}}$$

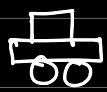
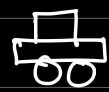
u = initial velocity

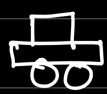
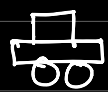
v = final velocity.

a = acceleration.

t = time.

UNITS : m/s^2 or ms^{-2}

(A) $u=0$  $t=4\text{ sec}$ $v=100$  $a = \frac{100-0}{4} = 25ms^{-2}$

(B) $u=0$  $t=10\text{ sec}$ $v=100$  $a = \frac{100-0}{10} = 10ms^{-2}$

Acc $\longrightarrow$ +ve $\longrightarrow$ Body is SPEEDING UP $\longrightarrow$ ACCELERATION

Acc $\longrightarrow$ −ve $\longrightarrow$ Body is SLOWING DOWN $\longrightarrow$ RETARDATION DECELERATION.

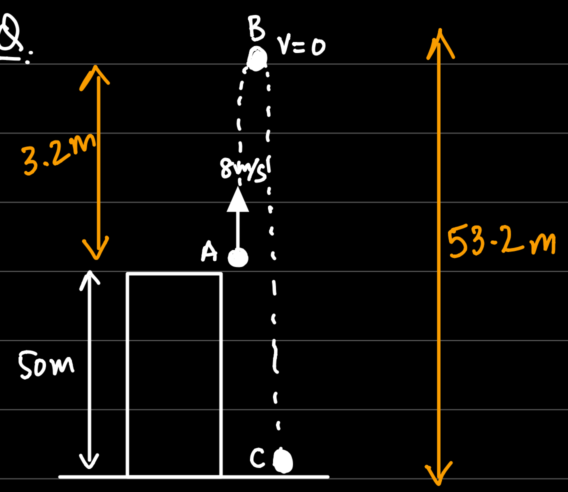
> +/− SIGN OF ACCELERATION CANNOT COMMENT ON DIRECTION OF MOTION OF BODY.

TYPE 1: CONSTANT ACCELERATION

CONSTANT SPEED	CONSTANT (UNIFORM) ACCELERATION.
$(a = 0)$	1) INCLINED PLANE 2) PULLEY (in any shape) 3) FREEFALL: (a) body is in air (b) only force acting on body is weight. $a = 10$ (speed up) or $a = -10$ (slowdown)
$\underset{\text{distance}}{d} = \underset{\text{speed}}{S} \times \underset{\text{time}}{t}$ $\underset{\text{Displacement}}{S} = \underset{\text{velocity}}{V} \times \underset{\text{time}}{t}$	$v = u + at$ $2as = v^2 - u^2$ $S = ut + \frac{1}{2}at^2$

v = final velocity
u = initial velocity
t = time
s = displacement.
a = acceleration.

Q.



A ball is thrown upwards from top of 50m tall building with 8m/s.

Find:

(i) max height reached by ball.

$$A \xrightarrow{\text{freefall.}} B$$

$u = 8, \quad a = -10, \quad v = 0$

$$2as = v^2 - u^2$$

$$2(-10)(s) = 0^2 - 8^2$$

$$s = 3.2\text{ m}$$

Max height $= 50 + 3.2 = 53.2$ m.

(ii) Velocity with which ball hits ground.

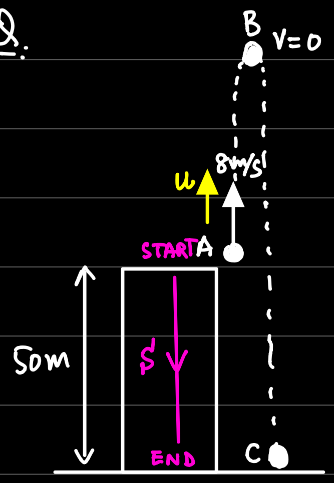
$$B \xrightarrow{\text{free fall}} C$$

$u = 0$, $a = +10$, $v = ?$, $S = 53.2$

$$2as = v^2 - u^2$$
$$2(10)(53.2) = v^2 - 0^2$$
$$v^2 = 1064$$
$$v = \sqrt{1064} = 32.6 \text{ m/s.}$$

IN M1, examiner requires to calculate from complete Journey from $A \longrightarrow C$.

Q.



A ball is thrown upwards from top of 50m tall building with 8m/s.

Find velocity of ball just before hitting ground.

$$A \xrightarrow{\text{FREEFALL}} C$$

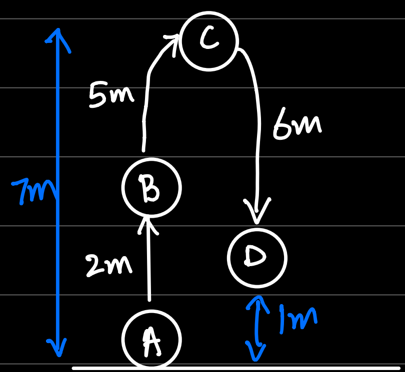
$u = 8$, $a = -10$, $S = -50$, $v = ?$

$$2as = v^2 - u^2$$
$$2(-10)(-50) = v^2 - 8^2$$
$$1000 = v^2 - 64$$
$$v^2 = 1064, \quad v = 32.6.$$

+/- SIGNS

ACCELERATION	DISPLACEMENT
STAND AT intialvelocity (u)	Mark an arrow from START → END and Label it (S).
IF BODY is ABOUT TO Speed up → a = +ve Slow down → a = -ve	Now compare this arrow with initial velocity (u) If both arrows are in: same direction → S = +ve opp direction → S = -ve.

IF A BODY STARTS FROM GROUND AND MOVES VERTICALLY, ITS DISPLACEMENT AND HEIGHT IS ALWAYS SAME

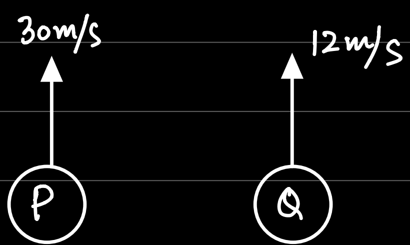


	Height	Displacent.
A	0	0
B	2	2
C	7	7
D	1	1

TIME DELAY BETWEEN TWO OBJECTS

In equations of motion, time of motion is needed.

$t_p = 5$ $\quad$ $t_q = 5-2 = 3$ $\quad$ $t = 5$ sec (Pause).



Q is released 2 seconds after P.

time $= t$ $\longrightarrow$ time $= t-2$

time $= T+2$ $\longleftarrow$ time $= T$

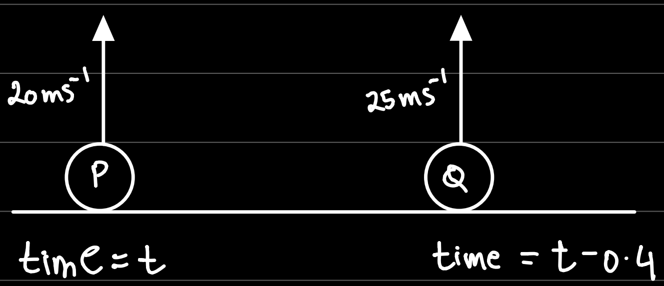
HEIGHT = DISPLACEMENT.

27 Particles P and Q are projected vertically upwards, from different points on horizontal ground, with velocities of $20\ \mathrm{m\,s^{-1}}$ and $25\ \mathrm{m\,s^{-1}}$ respectively. $\boxed{Q \text{ is projected } 0.4\text{ s later than } P.}$ Find

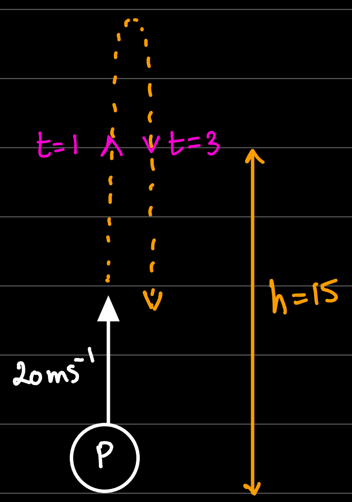
(i) the time for which P's height above the ground is greater than 15 m, [3]

(ii) the velocities of P and Q at the instant when the particles are at the same height. [5]

Same displacement.



(i)



$u = 20 \quad , \quad a = -10$

$s = 15 \quad , \quad t = ?$

$s = ut + \frac{1}{2}at^2$

$15 = 20t + \frac{1}{2}(-10)t^2$

$15 = 20t - 5t^2$

$5t^2 - 20t + 15 = 0$

$t^2 - 4t + 3 = 0$

$t^2 - 3t - t + 3 = 0$

$t(t-3) - 1(t-3) = 0$

$t = 1 \quad , \quad t = 3$

Time for which body stayed above 15m height $= 3 - 1 = 2\text{ sec}$

(ii) DISPLACEMENTS (HEIGHTS)

h_P = height of P
h_Q = height of Q.

P

$u = 20$, $a = -10$

$s = h_P$, time $= t$

$$s = ut + \frac{1}{2}at^2$$

$$h_P = 20t + \frac{1}{2}(-10)t^2$$

$$\boxed{h_P = 20t - 5t^2}$$

Q

$u = 25$, $a = -10$

$s = h_Q$, time $= t - 0.4$

$$s = ut + \frac{1}{2}at^2$$

$$h_Q = 25(t-0.4) + \frac{1}{2}(-10)(t-0.4)^2$$

$$\boxed{h_Q = 25(t-0.4) - 5(t-0.4)^2}$$

$$h_P = h_Q$$

$$20t - 5t^2 = 25(t-0.4) - 5(t-0.4)^2$$

$$20t - 5t^2 = 25t - 10 - 5(t^2 - 0.8t + 0.16)$$

$$20t - \cancel{5t^2} = 25t - 10 - \cancel{5t^2} + 4t - 0.8$$

$$0 = 9t - 10.8$$

$$t = 1.2$$

VELOCITIES

P

$v = u + at$

$v_P = 20 + (-10)(1.2)$

$v_P = 8$ m/s

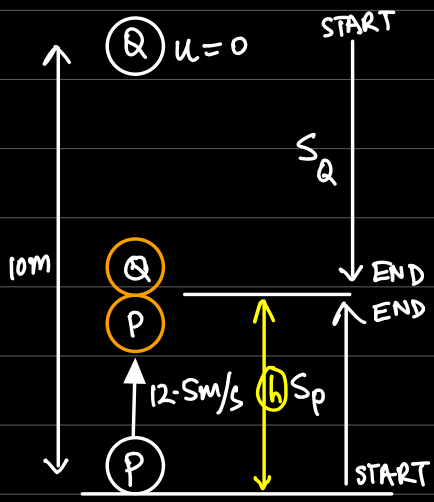
Q

$v = u + a(t-0.4)$

$v_Q = 25 + (-10)(1.2-0.4)$

$v_Q = 17$ m/s

21 A particle is projected vertically upwards from a point O with initial speed $12.5\,\mathrm{m\,s^{-1}}$. At the same instant another particle is released from rest at a point 10 m vertically above O. Find the height above O at which the particles meet. [5]



P	Q
$u = 12.5$, $a = -10$	$u = 0$, $a = +10$
$s = S_P$, time $= t$	$s = S_Q$, time $= t$
$s = ut + \frac{1}{2}at^2$	$s = ut + \frac{1}{2}at^2$
$S_P = (12.5)t + \frac{1}{2}(-10)t^2$	$S_Q = 0t + \frac{1}{2}(10)t^2$
$\boxed{S_P = 12.5t - 5t^2}$	$\boxed{S_Q = 5t^2}$

$$S_P + S_Q = 10$$

$$12.5t - 5t^2 + 5t^2 = 10$$

$$12.5t = 10$$

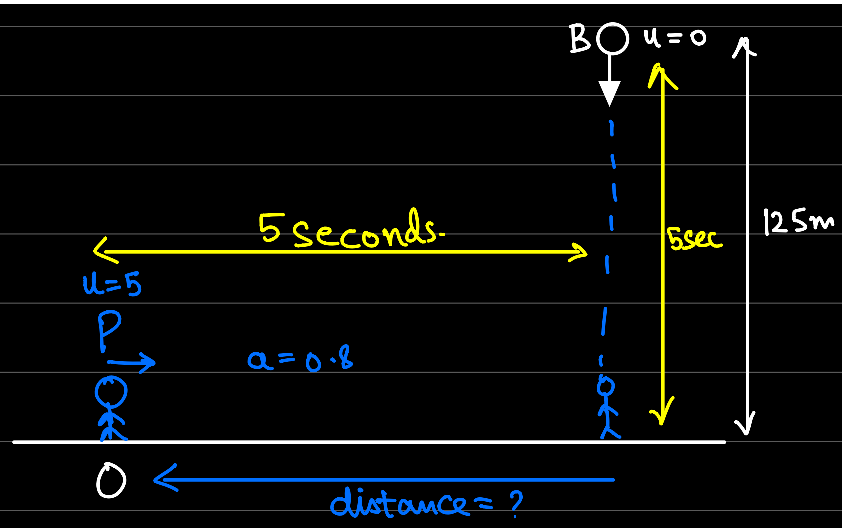
$$t = 0.8$$

$$h = S_P = 12.5t - 5t^2$$

$$= 12.5(0.8) - 5(0.8)^2$$

$$= 6.8\text{ m.}$$

51 An object is released from rest at a height of 125 m above horizontal ground and falls freely under gravity, hitting a moving target P. The target P is moving on the ground in a straight line, with constant acceleration $0.8\,\mathrm{m\,s^{-2}}$. At the instant the object is released P passes through a point O with speed $5\,\mathrm{m\,s^{-1}}$. Find the distance from O to the point where P is hit by the object. [4]



$\boxed{B}$ $u=0$, $s=125$, $t=?$, $a=+10$

$s = ut + \frac{1}{2}at^2$

$125 = 0t + \frac{1}{2}(10)t^2$

$125 = 5t^2$

$\boxed{t=5}$

$\boxed{P}$ $u=5$

$a=0.8$

$t=5$

$s=?$

$s = ut + \frac{1}{2}at^2$

$= 5(5) + \frac{1}{2}(0.8)(5)^2$

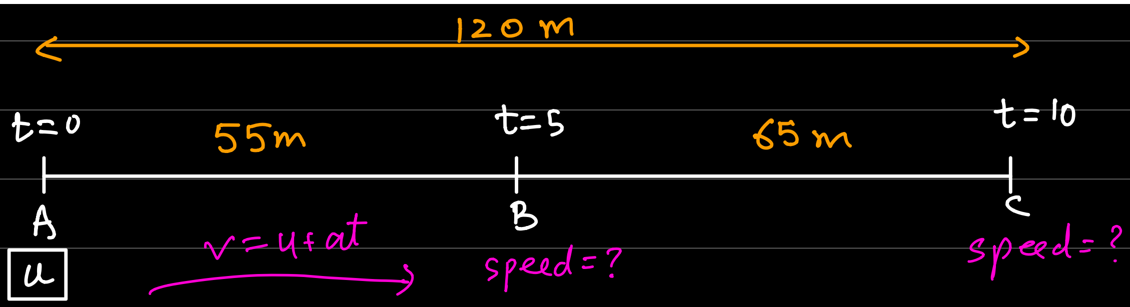
$s = 35\,\text{m}.$

$v = u + at$

$2as = v^2 - u^2$

$s = ut + \frac{1}{2}at^2$

54 A car travels along a straight road with constant acceleration a m s^{-2}. It passes through points A, B and C; the time taken from A to B and from B to C is 5 s in each case. The speed of the car at A is u m s^{-1} and the distances AB and BC are 55 m and 65 m respectively. Find the values of a and u. [6]



$A \longrightarrow B$

$u = u$, $a = a$

$t = 5$, $s = 55$

$$s = ut + \frac{1}{2}at^2$$

$$55 = u(5) + \frac{1}{2}a(5)^2$$

$$\boxed{55 = 5u + 12.5a}$$

$A \longrightarrow C$

$u = u$, $a = a$

$t = 10$, $s = 120$

$$s = ut + \frac{1}{2}at^2$$

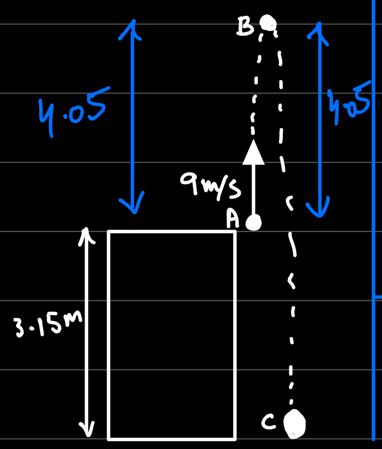
$$120 = u(10) + \frac{1}{2}(a)(10)^2$$

$$\boxed{120 = 10u + 50a}$$

$u =$

$a =$

71 A particle is projected vertically upwards with speed 9 m s^{-1} from a point 3.15 m above horizontal ground. The particle moves freely under gravity until it hits the ground. For the particle's motion from the instant of projection until the particle hits the ground, find the total distance travelled and the total time taken. [6]



$A \longrightarrow B$

$u = 9, \; a = -10$

$s = ?\;, \; v = 0$

$2as = v^2 - u^2$

$2(-10)s = 0^2 - 9^2$

$s = 4.05$

$v = u + at$

$0 = 9 + (-10)t$

$t = 0.9$

Total Distance

$4.05 + 4.05 + 3.15$

$= 11.25\,m.$

$B \longrightarrow C$

$u = 0, \; t = ?, \; a = 10$

$s = 4.05 + 3.15 = 7.2$

$s = ut + \frac{1}{2}at^2$

$7.2 = 0t + \frac{1}{2}(10)t^2$

$\boxed{t = 1.2}$

Direct: $A \longrightarrow C$

$u = 9, \; a = -10$

$s = -3.15, \; t = ?$

$s = ut + \frac{1}{2}at^2$

$-3.15 = 9t + \frac{1}{2}(-10)t^2$

Solve Quadratic.

Total time $= 1.2 + 0.9 = 2.1$

TYPE 2 : NON CONSTANT ACCELERATION

$$\boxed{\text{DISPLACEMENT}} \underset{\text{integrate}}{\overset{\text{DIFF}}{\rightleftarrows}} \boxed{\text{VELOCITY}} \underset{\text{integrate}}{\overset{\text{DIFF}}{\rightleftarrows}} \boxed{\text{ACCELERATION}}$$

CASE 1:

VALUE OF DISPLACEMENT IS $\boxed{\text{GIVEN}}$ OR $\boxed{\text{TO BE FOUND}}$

INTEGRATE WITH LIMITS.

CASE 2:

Expression for Displacement

INTEGRATE WITHOUT LIMITS

USE $\boxed{+C}$

For integrating from acceleration to velocity, ALWAYS integrate "WITHOUT LIMITS" and use $\boxed{+C}$

$$s = 6t^3 - 7t^2 + 12$$

Diff ↓ ↑ integrate.

$$v = 18t^2 - 14t$$

Diff ↓ ↑ integrate

$$a = 36t - 14$$

TURNING POINT (INSTANTENOUS REST)		$V=0$
MAX DISPLACEMENT	disp $\xrightarrow{\text{diff}}$ velocity	$V=0$
MAX VELOCITY	velocity $\xrightarrow{\text{Diff}}$ acc	$a=0$
MAX ACCELERATION	acc $\xrightarrow{\text{diff}}$ $\frac{da}{dt}$	$\frac{da}{dt}=0$

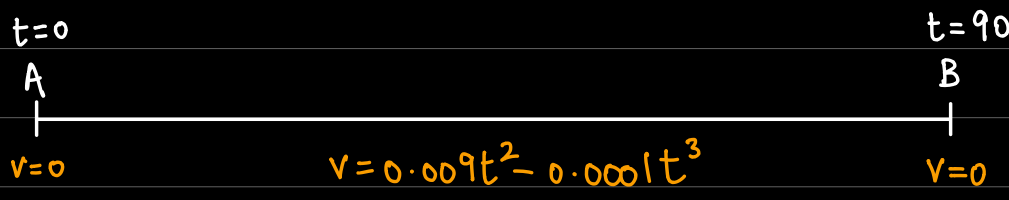
$t=0$ $v=0$

2 A particle starts from rest at the point A and travels in a $\boxed{\text{straight line}}$ until it reaches the point B. The velocity of the particle t seconds after leaving A is $v \text{ m s}^{-1}$, where $v = 0.009t^2 - 0.0001t^3$. Given that the velocity of the particle when it reaches B is zero, find

(i) the time taken for the particle to travel from A to B, [2]

(ii) the distance AB, (value) (with limits Integration). [4]

(iii) the maximum velocity of the particle. (acc = 0) [4]



(i) Put $v=0$

$$0 = 0.009t^2 - 0.0001t^3$$

$$0 = t^2(0.009 - 0.0001t)$$

$$t^2 = 0 \quad , \quad 0.009 - 0.0001t = 0$$

$$t = 0 \qquad\qquad t = 90$$

A B

(ii) Integration. (with limits)

$$s = \int_0^{90} (0.009t^2 - 0.0001t^3)\, dt$$

$$s = \left| \frac{0.009t^3}{3} - \frac{0.0001t^4}{4} \right|_0^{90}$$

$$s = \left| \left(\frac{0.009(90)^3}{3} - \frac{0.0001(90)^4}{4} \right) - \left(\frac{0.009(0)^3}{3} - \frac{0.0001(0)^4}{4} \right) \right|$$

$$s = 546.75$$

(iii) Max velocity $a = 0$

$v = 0.009t^2 - 0.0001t^3$	$v = 0.009(60)^2 - 0.0001(60)^3$
↓ diff	
$a = 0.018t - 0.0003t^2$	$v = 10.8$ m/s
$0 = 0.018t - 0.0003t^2$	
$0.0003t^2 = 0.018t$	
$0.0003t = 0.018$	
$t = 60$	

40 A particle travels in a straight line from A to B in 20 s. Its acceleration t seconds after leaving A is $a\ \mathrm{m\,s^{-2}}$, where $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$. It is given that the particle comes to rest at B.

(i) Show that the initial speed of the particle is zero. [4]

$t = 0$

(ii) Find the maximum speed of the particle. (put acc = 0) [2]

(iii) Find the distance AB. (integrate velocity) (with limits) [4]

$t=0$ $t=20$

A $a = \frac{3}{160}t^2 - \frac{1}{800}t^3$ B

$v = ?$ $v = 0$

(i) integrate without limits (+c)

$$v = \int \left(\frac{3}{160}t^2 - \frac{1}{800}t^3 \right) dt .$$

$$v = \frac{3}{160}\frac{t^3}{3} - \frac{1}{800}\frac{t^4}{4} + C$$

$$\boxed{v = \frac{1}{160}t^3 - \frac{1}{3200}t^4 + c}$$

$v = 0$, $t = 20$ at B

$$0 = \frac{1}{160}(20)^3 - \frac{1}{3200}(20)^4 + c$$

$$c = 0$$

$$\boxed{v = \frac{1}{160}t^3 - \frac{1}{3200}t^4}$$

For initial velocity, put $t=0$

$$v = \frac{1}{160}(0)^3 - \frac{1}{3200}(0)^4 = 0 .$$

(ii) Max Speed (acc = 0)

$$a = \frac{3}{160}t^2 - \frac{1}{800}t^3$$

$$0 = \frac{3}{160}t^2 - \frac{1}{800}t^3$$

$$0 = t^2\left(\frac{3}{160} - \frac{1}{800}t\right)$$

$$\frac{3}{160} - \frac{1}{800}t = 0$$

$$t = 15$$

max velocity, put $t = 15$

$$V = \frac{1}{160}t^3 - \frac{1}{3200}t^4$$

$$V = \frac{1}{160}(15)^3 - \frac{1}{3200}(15)^4$$

$$V = 5.27.$$

$$S = \int_0^{20}\left(\frac{1}{160}t^3 - \frac{1}{3200}t^4\right)dt$$

$$\left|\frac{1}{160}\frac{t^4}{4} - \frac{1}{3200}\frac{t^5}{5}\right|_0^{20}$$

$$\left|\frac{t^4}{640} - \frac{t^5}{16000}\right|_0^{20}$$

$$\left|\left(\frac{20^4}{640} - \frac{20^5}{16000}\right) - \left(\frac{0^4}{640} - \frac{t^5}{16000}\right)\right|$$

$$= 50.$$

TYPE 2 ADVANCED SPLIT JOURNEY.

$t=0$ — $t=15$ — $t \geq 15$

$v = 3t^2 - 9t$ (from $t=0$ to $t=15$) $v = \frac{B}{t^3}$ (for $t \geq 15$)

	AT BOUNDARY TIME
INSTANTENEOUS DISP. STAYS SAME	1- Displacement is same in both equation
INSTANTENEOUS VELOCITY STAYS SAME	2- Velocity is same from both expressions
INSTANTENEOUS ACCELERATION MAY OR MAY NOT BE SAME.	3- Acc may or may not be same.

Find Value of B:

put $t=15$ in both velocity expressions and equate them.

$v = 3t^2 - 9t$ $\boxed{t=15}$ $v = \frac{B}{t^3}$

$$3(15)^2 - 9(15) = \frac{B}{15^3}$$

$$B = 1822500.$$

21 A vehicle is moving in a straight line. The velocity v m s^{-1} at time t s after the vehicle starts is given by

$$v = A(t - 0.05t^2) \quad \text{for } 0 \leqslant t \leqslant 15,$$
$$v = \frac{B}{t^2} \quad \text{for } t \geqslant 15,$$

where A and B are constants. The distance travelled by the vehicle between $t = 0$ and $t = 15$ is 225 m.

(i) Find the value of A and show that $B = 3375$. [5]

$v = \dfrac{3375}{t^2}$

without limits. integration

(ii) Find an expression in terms of t for the total distance travelled by the vehicle when $t \geqslant 15$. [3]

(iii) Find the speed of the vehicle when it has travelled a total distance of 315 m. [3]

$t=0$ $\quad$ $S = 225$, $t = 15$ $\quad$ $t > 15$

$v = A(t - 0.05t^2)$, $A = 4$ $\quad$ $v = \dfrac{B}{t^2}$, $B = 3375$

(i) $$S = \int A(t - 0.05t^2)\,dt$$

$$225 = \int_0^{15} A(t - 0.05t^2)\,dt$$

$$225 = A\int_0^{15} (t - 0.05t^2)\,dt$$

$$225 = A\left|\frac{t^2}{2} - \frac{0.05t^3}{3}\right|_0^{15}$$

$$225 = A\left|\left(\frac{15^2}{2} - \frac{0.05(15)^3}{3}\right) - \left(\frac{0^2}{2} - \frac{0.05(0)^3}{3}\right)\right|$$

$$225 = A\left|\frac{225}{4}\right|$$

$$225 = A\left(\frac{225}{4}\right)$$

$$A = 4$$

FOR B we use BOUNDARY TIME

$t = 15$

$$v = 4(t - 0.05t^2) \qquad v = \frac{B}{t^2}$$

$$4(15 - 0.05(15)^2) = \frac{B}{15^2}$$

$$15 = \frac{B}{225}$$

$$B = 3375$$

(ii)

$$s = \int \frac{3375}{t^2}\,dt$$

$$s = \int 3375t^{-2}\,dt$$

$$s = 3375\frac{t^{-1}}{-1} + c$$

$$\boxed{s = -\frac{3375}{t} + c}$$

This c will always be calculated from the displacement and time of BOUNDARY TIME.

$s = 225$, $t = 15$

$$225 = -\frac{3375}{15} + c$$

$$c = 450$$

$$\boxed{s = -\frac{3375}{t} + 450}$$

Expression for total distance travelled.

(iii) Total distance = 315

$$S = -\frac{3375}{t} + 450$$

$$315 = \frac{-3375}{t} + 450$$

$$\frac{3375}{t} = 450 - 315$$

$$t = 25$$

For speed put $t = 25$ in:

$$v = \frac{3375}{t^2}$$

$$v = \frac{3375}{25^2}$$

$$v = 5.4.$$

5 A particle P starts from rest at O and travels in a straight line. Its velocity $v\ \mathrm{m\,s^{-1}}$ at time t s is given by $v = 8t - 2t^2$ for $0 \leqslant t \leqslant 3$, and $v = \frac{54}{t^2}$ for $t > 3$. Find

$t=0$ $v=0$

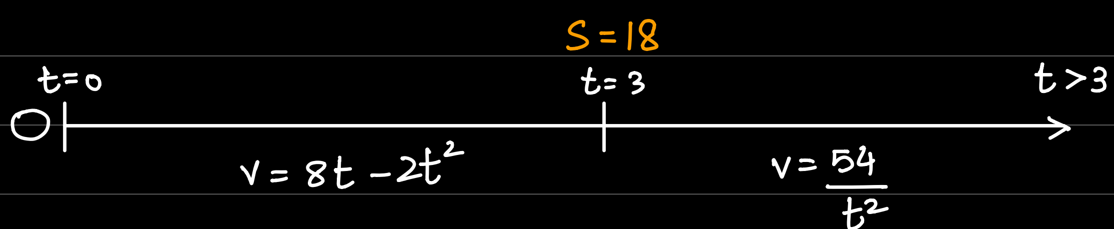
integrate with limits.

(i) the distance travelled by P in the first 3 seconds, [4]

without limits. integrate

(ii) an expression in terms of t for the displacement of P from O, valid for $t > 3$, [3]

(iii) the value of v when the displacement of P from O is 27 m. [3]



(i)
$$S = \int_0^3 (8t - 2t^2)\,dt$$

$$S = \left|\frac{8t^2}{2} - \frac{2t^3}{3}\right|_0^3$$

$$S = \left|4t^2 - \frac{2t^3}{3}\right|_0^3$$

$$s = \left| \left(4(3)^2 - \frac{2(3)^3}{3} \right) - \left(4(0)^2 - \frac{2(0)^3}{3} \right) \right|$$

$s = 18m$.

(ii) $$s = \int \frac{54}{t^2} \, dt$$

$$s = \int 54t^{-2} dt$$

$$s = \frac{54t^{-1}}{-1} + c$$

$$\boxed{s = -\frac{54}{t} + c}$$

$\left.\begin{matrix} t = 3 \\ s = 18 \end{matrix}\right\}$ BOUNDARY TIME.

$$18 = -\frac{54}{3} + c$$

$$c = 36$$

$$\boxed{s = \frac{-54}{t} + 36}$$

(iii) $s = 27$

$$s = \frac{-54}{t} + 36$$

$$27 = -\frac{54}{t} + 36$$

$$\frac{54}{t} = 36 - 27$$

$$\boxed{t = 6}$$

$$v = \frac{54}{t^2}$$

$$v = \frac{54}{6^2}$$

$$v = 1.5$$

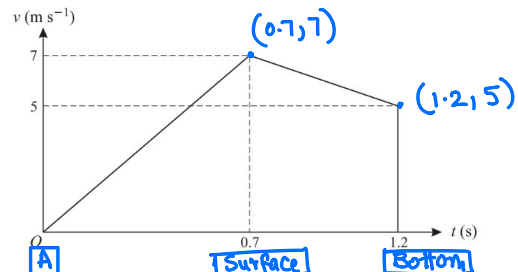
TYPE 3 GRAPHS

DISPLACEMENT $\xrightarrow{\text{GRADIENT}}$ VELOCITY $\xrightarrow{\text{GRADIENT}}$ ACCELERATION

ACCELERATION $\xrightarrow{\text{Area}}$ VELOCITY $\xrightarrow{\text{Area}}$ DISPLACEMENT

IMP: ALWAYS DRAW REAL LIFE DIAGRAM OF SCENARIO AND RELATE KEY TIMES TO GRAPH.

7



The diagram shows the velocity-time graph for the motion of a small stone which falls vertically from rest at a point A above the surface of liquid in a container. The downward velocity of the stone t s after leaving A is v m s^{-1}. The stone hits the surface of the liquid with velocity 7 m s^{-1} when $t = 0.7$. It reaches the bottom of the container with velocity 5 m s^{-1} when $t = 1.2$.

(i) Find

Displacement (area under graph)

(a) the height of A above the surface of the liquid, (From A to Surface)

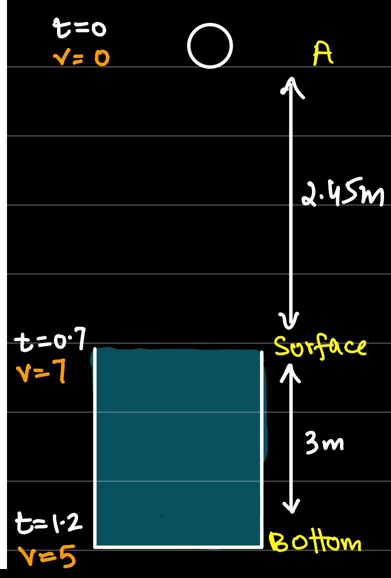
(b) the depth of liquid in the container. [3]

Displacement (area under graph)

(ii) Find the deceleration of the stone while it is moving in the liquid. [2]

acc (grad) when slowing down

(iii) Given that the resistance to motion of the stone while it is moving in the liquid has magnitude 0.7 N, find the mass of the stone. [3]



(i) height of A to Surface

$$\text{Area} = \frac{1}{2}(0.7)(7) = 2.45\text{m}$$

(ii) Depth = Area from Surface to Bottom

$$\text{Area of trapezium} = \frac{1}{2} \times h \times (a+b)$$

$= \frac{1}{2} \times 0.5 \times (7+5)$

$= 3\text{ m}$

(iii) Deceleration: $(0.7, 7)$ $(1.2, 5)$

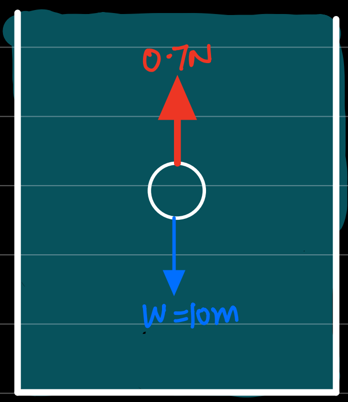
grad $=$ acc $= \dfrac{5-7}{1.2-0.7} = \dfrac{-2}{0.5} = -4\text{ m/s}^2$

acc $= -4$

Deceleration $= 4\text{ m/s}^2$

(iii)

(iii) Given that the resistance to motion of the stone while it is moving in the liquid has magnitude 0.7 N, find the mass of the stone. [3]



Fwd − Bwd $= ma$ acc

$10m - 0.7 = m(-4)$

$10m + 4m = 0.7$

$14m = 0.7$

$m = 0.05$

$W = mg = m(10) = 10m$

$\boxed{\text{ACC}}$

KINEMATICS

$v = u + at$

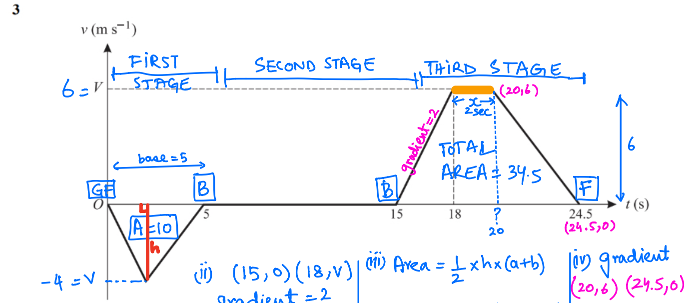
$2as = v^2 - u^2$

$s = ut + \frac{1}{2}at^2$

FORCES

$Fwd - Bwd = ma$

3



(i) $A = \frac{1}{2} \times b \times h$

$10 = \frac{1}{2} \times (5) \times h$

$h = 4$

speed $= 4$

(ii) $(15, 0)\ (18, V)$

gradient $= 2$

$\frac{V - 0}{18 - 15} = 2$

$V = 6$

(iii) Area $= \frac{1}{2} \times h \times (a + b)$

$34.5 = \frac{1}{2} \times 6 \times (9.5 + x)$

$69 = 6(9.5 + x)$

$x = 2$

(iv) gradient

$(20, 6)\ (24.5, 0)$

$a = \frac{0 - 6}{24.5 - 20}$

$a = -\frac{4}{3}$

dec $= \frac{4}{3}$.

The diagram shows the velocity-time graph for a lift moving between f oors in a building. The graph consists of straight line segments. In the f rst stage the lift travels downwards from the ground f oor for 5 s, coming to rest at the basement after travelling 10 m. (area = 10)

(i) Find the greatest speed reached during this stage. [2]

The second stage consists of a 10 s wait at the basement. In the third stage, the lift travels upwards until it comes to rest at a floo 34.5 m above the basement, arriving 24.5 s after the start of the fi st stage. The lift accelerates at 2 m s^{-2} for the fi st 3 s of the third stage, reaching a speed of V m s^{-1}. Find

gradient = 2

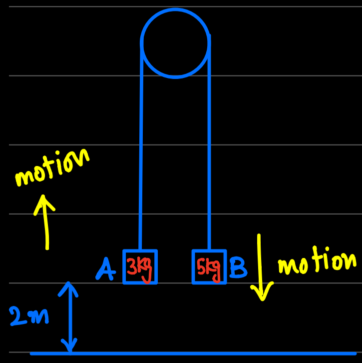
(ii) the value of V, [2]

(iii) the time during the third stage for which the lift is moving at constant speed, [3]

(iv) the deceleration of the lift in the fina part of the third stage. [2]

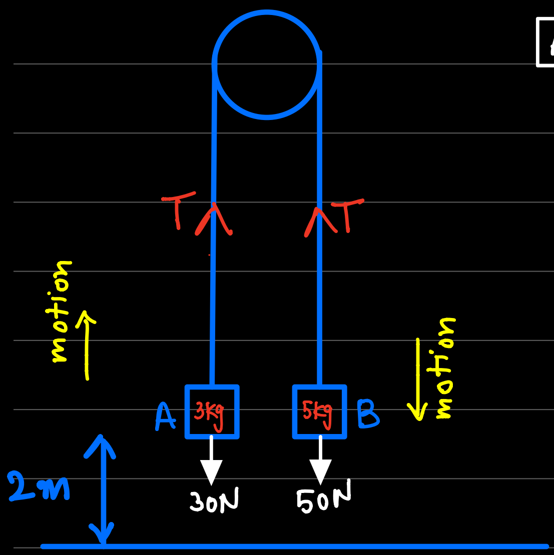
gradient.

Pulleys



Mass of 3kg and 5kg are held at rest. The system is released and objects start to move.

(i) Find the acceleration of both particles and tension in string.



$\boxed{A}$ Fwd − Bwd = ma

$T - 30 = 3a$

$T = 30 + 3a$

$\boxed{B}$ Fwd − Bwd = ma

$50 - T = 5a$

$T = 50 - 5a$

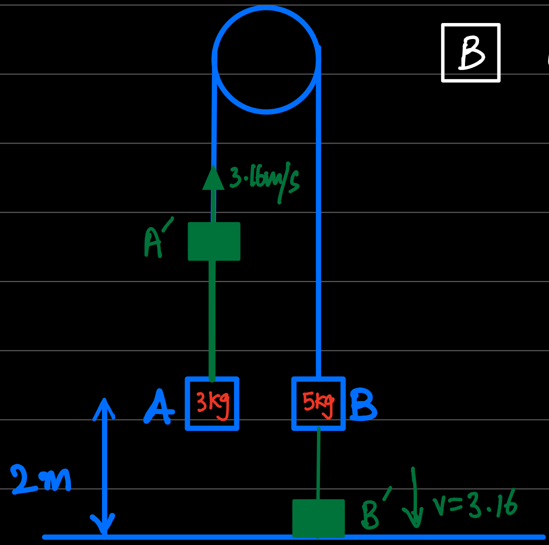
$30 + 3a = 50 - 5a$

$8a = 20$

$a = 2.5\ m/s^2$

$T = 30 + 3(2.5) = 37.5\ N.$

(ii) Find speed with which B hits ground.



$\boxed{B}$ $u = 0$, $v = ?$, $s = 2$, $a = 2.5$

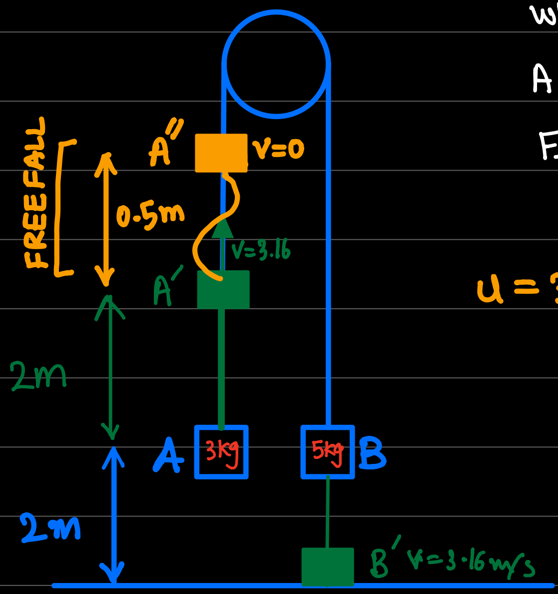
$2as = v^2 - u^2$

$2(2.5)(2) = v^2 - 0^2$

$10 = v^2$

$v = \sqrt{10} = 3.16$

(iii) Find greatest height above ground reached by A (5)



When B hits ground it comes to rest

A keeps moving up, under gravity

Freefall because string is not taut

A' —— A''

$u = 3.16,\ s = ?,\ a = -10,\ v = 0$

$$2as = v^2 - u^2$$

$$2(-10)s = 0^2 - 3.16^2$$

$$s = 0.5$$

$\boxed{\text{VARIATIONS}}$

Find:

(a) Max height above ground reached by A.

$2 + 2 + 0.5 = 4.5\text{ m}$

(b) Distance travelled by A to reach max height.

$2 + 0.5 = 2.5\text{ m}$

(c) Distance travelled by A from start of journey till it comes to rest again.

$2 + 0.5 + 0.5 = 3\text{ m}$